DOM

Unit I : 1) Friction & Friction Clutches

2) Brakes & Dynamometers

Friction Clutches Unit-II

A clutch is a device used to transmit the rotary motion of one shaft to another when desired. The axes of the two shafts are coincident.

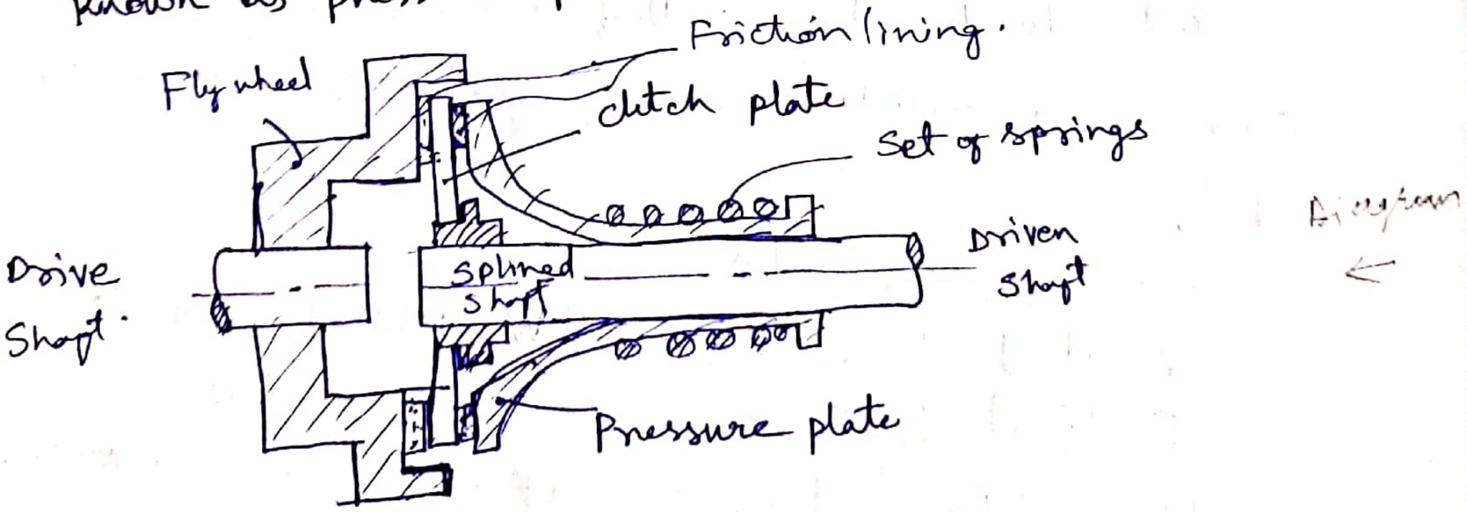
In friction clutches, the connection of the engine shaft to the gear-box is affected by friction between two ~~shaft~~ or more rotating concentric surfaces can be pressed firmly against one another when engaged and the clutch tends to rotate as a single unit.

Types of friction clutches:-

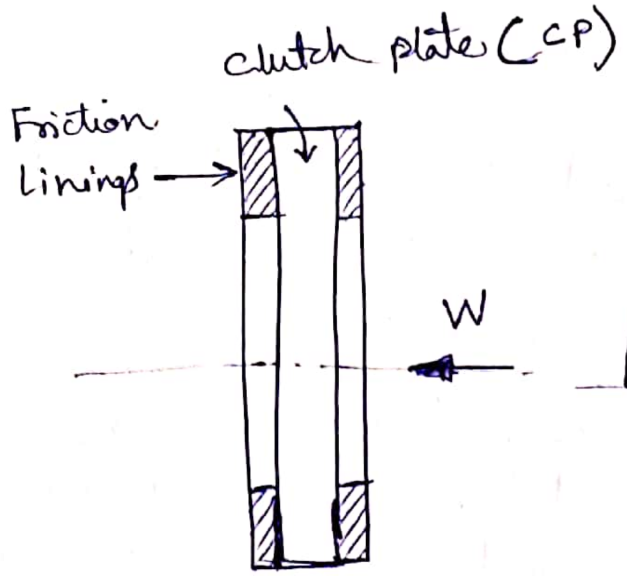
1) Disc clutch (Single-Plate clutch).
2) Multiplate clutch.
3) Cone clutch.
4) Centrifugal clutch.

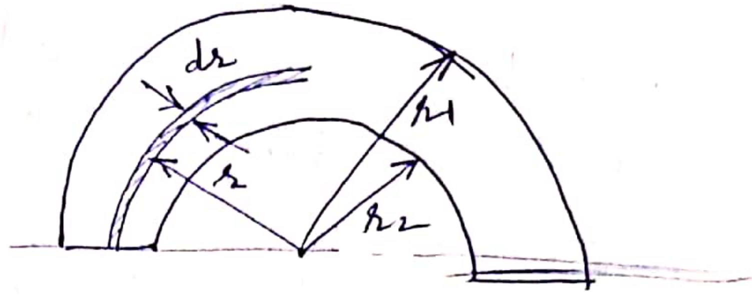
1) Single-Plate Clutch:

All modern cars have single plate clutch with a fabric facing on each side of the plate. The clutch plate is positioned between the flywheel and a Solid plate known as pressure plate.



clutch in engaged position.





✓ Let r_1 = External radius of friction lining on clutch plate. (CP)

r_2 = Internal radius of " " "

p = Intensity of pressure on CP by pressure plate.

W = Total axial load (Axial thrust)

μ = coefficient of friction.

T = Torque transmitted.

✓ Principle of single plate clutch:- The power transmitted by friction ~~of to~~ linings should be more & hence the coeff. of friction should be more.

On New clutch plates → The intensity of pressure is uniform over the entire surface.

Old clutch plates → Uniform wear theory is applied.

Consider a circular ring of radius "r" and thickness "dr".

Area of ring $= dA = 2\pi r \cdot dr$.

Axial load on ring $= dW = p \cdot dA = p \cdot 2\pi r dr$

Frictional force on ring $= dF = \mu \cdot dW$

dT = Frictional torque $= dF \times r = \mu dW r = \mu p \pi r \cdot 2 \cdot dr \cdot r$

$$dT = \mu p 2\pi r^2 dr$$

There are two criteria to obtain the torque transmission capacity of friction clutches i.e Uniform pressure & Uniform wear.

(i) For new clutches (Uniform pressure); $p = \dfrac{W}{\pi(r_1^2 - r_2^2)}$

Frictional torque on one side of clutch plate $T = \int_{r_2}^{r_1} dT = \int_{r_2}^{r_1} 2\pi \mu p r^2 dr$

$$= 2\pi \mu p \left[\frac{r^3}{3}\right]_{r_2}^{r_1}$$

$$T = 2\pi \mu p \left(\frac{r_1^3 - r_2^3}{3}\right)$$

$$T = 2\pi \mu \left(\frac{W}{\pi(r_1^2 - r_2^2)}\right)\left(\frac{r_1^3 - r_2^3}{3}\right)$$

Frictional torque, $\boxed{T = \frac{2}{3} \mu W \left(\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right)}$ → New clutch (or) Uniform pressure.

Also $\boxed{T = \mu \cdot W \cdot R_m}$ where R_m = mean radius.

By comparing, $\boxed{R_m = \frac{2}{3}\left(\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right)}$

Total torque, $\boxed{T_{total} = 2T = 2 \times \frac{2}{3} W \mu \left(\frac{r_1^3 - r_1^3}{r_1^2 - r_2^2}\right)}$ ✓

In a single clutch plate there are two friction surfaces one on each side of the friction plate. Hence $T_{total} = 2T$.

(ii) Torque transmission under uniform wear (old C.P) :

In this ~~cas~~ wear is uniformly distributed over the entire surface area of friction disc.

Let $p \times r = \text{constant}$ → Inner (r_2) ∵ max. pr. will be on inner side

$$\boxed{p = \frac{c}{r}}$$

Axial load on ring, $dW = p \cdot 2\pi r \cdot dr$.

Total axial load $W = \int_{r_2}^{r_1} p \cdot 2\pi r \, dr = \int_{r_2}^{r_1} \frac{c}{\cancel{r}} 2\pi \cancel{r} \, dr$

$$W = 2\pi c \, r \Big|_{r_2}^{r_1} = 2\pi c (r_1 - r_2)$$

$$\boxed{c = \frac{W}{2\pi (r_1 - r_2)}}$$

Total frictional torque, $T = \int_{r_2}^{r_1} dT = \int_{r_2}^{r_1} \mu \cdot p \, 2\pi r^2 \, dr$
on one side of CP.

$$= \int_{r_2}^{r_1} \mu \cdot \frac{c}{\cancel{r}} \cdot 2\pi r^{\cancel{2}} \, dr = \mu c 2\pi \int_{r_2}^{r_1} r \, dr.$$

$$T = \mu c 2\pi \frac{r^2}{2} \Big|_{r_2}^{r_1} = \mu c \cancel{2}\pi \left(\frac{r_1^2 - r_2^2}{\cancel{2}} \right)$$

$$T = \mu c \pi (r_1^2 - r_2^2)$$

$$T = \mu \cdot \frac{W}{2\cancel{\pi} (r_1 - \cancel{r_2})} \cancel{\pi} (r_1 - \cancel{r_2})(r_1 + r_2) \quad [a^2 - b^2 = (a+b)(a-b)]$$

$$\boxed{T = \frac{\mu \cdot W}{2} (r_1 + r_2)} \boxed{= \mu W R_m}$$

$$R_m = \left(\frac{R_1 + R_2}{2} \right)$$

Total torque on single clutch plate (Both sides)

$$T_{total} = 2T = \cancel{2} \cdot \frac{\mu W}{\cancel{2}}(r_1 + r_2)$$

Total Torque, $\boxed{T_{total} = \mu W (r_1 + r_2)}$

The uniform pressure theory is applicable only when the friction lining is new. When the lining is used after period of time, the wear occurs, the major portion of the ~~plate~~ friction life comes under uniform wear criteria.

Hence in the design of clutches, the uniform wear theory is used.

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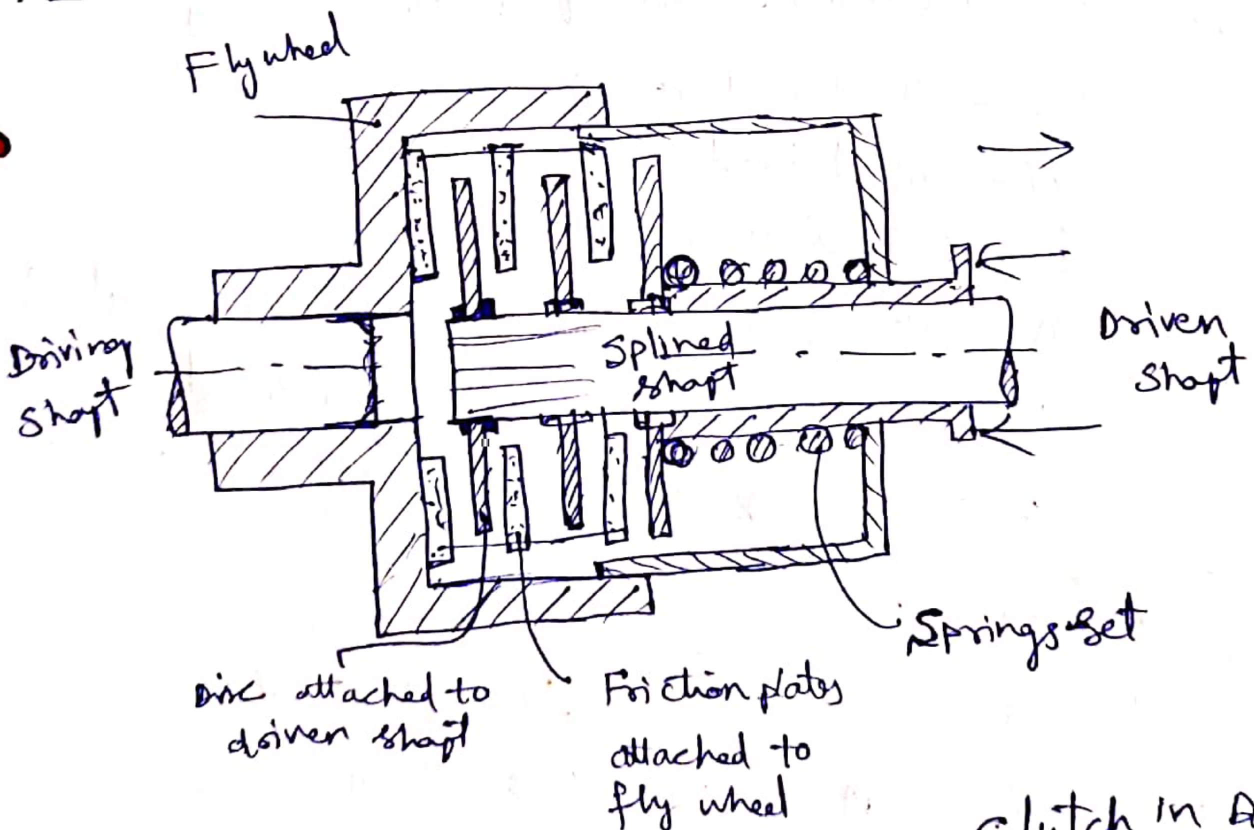
2) Multi plate clutch :-



Clutch in disengaged position.

The friction plates having friction linings on both sides except the first plate. This plate is having friction lining on one side. The friction plates are connected on top of the flywheel. Hence the friction plates rotates with the flywheel and thereby with the driven shaft on engagement. The friction plates are free to move axially like ~~dragon~~ disc move axially on driven shaft-on splines. The discs are located in b/w friction plates.

Multiplate clutch is used when large torque is to be transmitted, Incase of automobile & machinetools. Also for compact diameter of clutch plate where ever space restriction prevails.

Let r_1 = External radius of friction lining as friction plates

r_2 = Internal radius

W = Axial load

P = Intensity of pressure

n_1 = No. of friction plates on driving shaft/flywheel

n_2 = " discs on driven shaft.

n = no. of active surfaces i.e friction surfaces

$$\boxed{\therefore n = n_1 + n_2 - 1}$$

Total torque transmitted; $T = n \mu W R_m$

where R_m = Mean radius of friction surfaces.

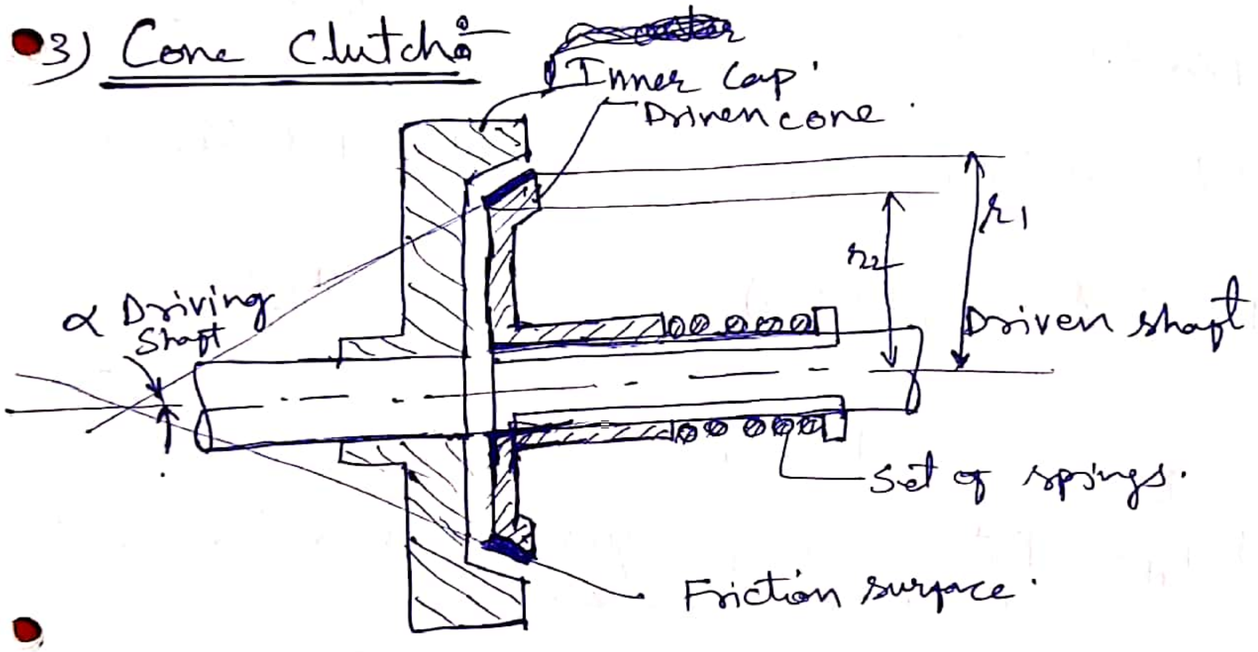
$$R_m = \frac{r_1 + r_2}{2}$$ for uniform wear theory.

$$R_m = \frac{2}{3}\left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]$$ for uniform pressure theory.

$$\boxed{W = 2\pi C (r_1 - r_2)}$$

$$\boxed{\text{Power, } P = \frac{2\pi N T}{60}}$$

3) Cone Clutch:



Let r_1 = External radius of conical friction surface.

r_2 = Internal radius of conical friction surface.

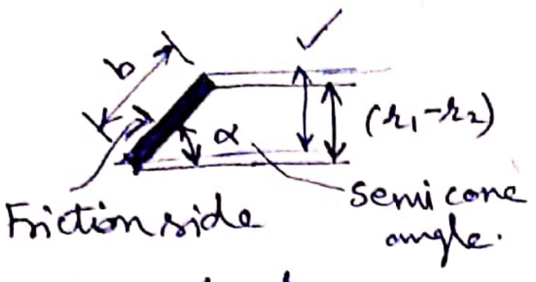
α = semi cone angle (or) the angle of the friction surface with the axis of the shaft.

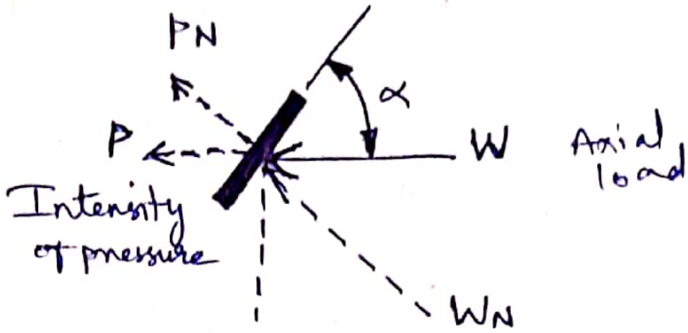
W = Total axil load (required to engage the clutch, supplied by spring)

R_M = Mean Radius of friction surface

μ = coefficient of friction

b = width of cone surface (or) contact surface.





$$b = \frac{r_1 - r_2}{\sin\alpha}$$

Uniform Wear	Uniform Pressure
$P \times r = \text{Constant } (c)$ $P_{max} \times r_2 = c$	$P = \dfrac{W}{\pi(r_1^2 - r_2^2)} = \text{constant}$
$W = 2\pi c\,(r_1 - r_2)$	$W = P \cdot \pi\,(r_1^2 - r_2^2)$
$T = \frac{1}{2}\,\mu \cdot \frac{W}{\sin\alpha}\,(r_1 + r_2)$	$T = \frac{2}{3}\,\mu\,\frac{W}{\sin\alpha}\left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]$

Driving torque based on mean radius :-

Let P_N = Intensity of pressure at mean radius normal to friction surface.

W_N = Total load normal to friction surface (Normal force)

$$W_N = \frac{W}{\sin\alpha}$$

W_N = pressure normal to friction surface × Area of friction surface based on mean radius

$$W_N = P_N \times 2\pi R_m \times b$$

$R_m = \left(\frac{r_1+r_2}{2}\right)$

$$W_N = W/\sin\alpha$$

$$\therefore T = \frac{1}{2}\,\mu W_N (r_1 + r_2)$$

$(\because T = \mu W_N R_m)$ Torque in terms of normal load & mean radius

$$\boxed{T = \frac{1}{2}\,\mu\,\frac{W}{\sin\alpha}\,R_m}$$

(Slope angle 8–15° Recomended)

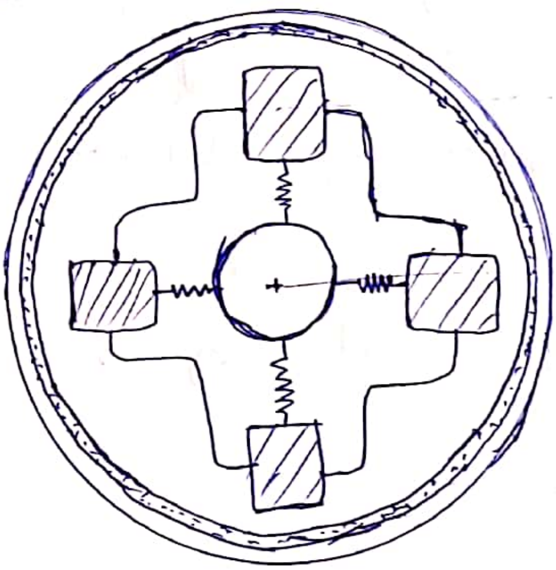
The slope of the cone is made small, that helps to give higher normal forces. The recomended angle of slope is in between 8–15°. According to the allowable normal pressure and coefficient of friction required at the contact face of the driven member is lined with material like lather, asbestos, wood etc.

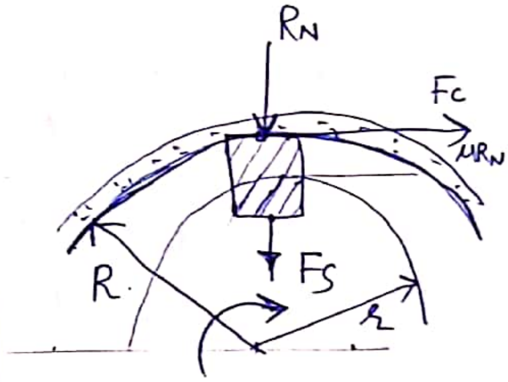
Advantages	Disadvantages
→ Small axial force is req. to keep the clutch engaged → Simple design → For a given dimension, the torque transmitted by cone clutch is higher than that of a single plate clutch	→ One pair of friction surface only → The tendency to grab the small cone angle causes some reluctance in dis engagement.

4) Centrifugal clutch :

Centrifugal force

Principle on





Let F_c = Centrifugal force due to mass of the sliding block.

F_S = Radial Spring force.

R_N = Reaction on the case due to F_c & F_s, (N)

R = Radius of the drum (m)

r = Radius of C.G of the sliding block, (m)

μR_N = Frictional force, (N)

μ = coefficient of friction

$F_c = m\omega^2 r$ (at full speed)

where, m = mass of sliding block/shoe (Kg)

ω = Angular velocity (rad/s)

$F_s = m\omega_1^2 r$ (At speed overcoming the resistⁿ of spring by centrifugal force at full speed)

$\omega_1 = x \cdot \omega$

where x = factor of angular velocity at which centrifugal force overcoming the spring force.

Frictional Torque, $T = \mu \cdot R_N \cdot R$ (where $R_N = F_c - F_s$)
on each shoe

Total torque, $T = n \cdot T$

where n = number of shoes.

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Single Plate clutch:-

① Calculate the power transmitted by a single plate clutch at a speed of 2000rpm. If the outer and Inner radii of friction ~~clutch~~ surface are 150mm & 100mm respectively. The max. intensity of pressure at any point of contact surface should not exceed $0.8 \times 10^5 \, N/m^2$. Take both sides of the plate as effective & coeff. of friction $= 0.3$. "Assume uniform wear".

Sol. Given, $N = 2000$ rpm, $r_1 = 150\,mm = 0.15\,m$

$P_{max} = 0.8 \times 10^5 \, N/m^2$ $r_2 = 100\,mm = 0.1\,m$

$\mu = 0.3$, No. of eff. sides $= 2$

For uniform wear, $P \times r = C \Rightarrow 0.8 \times 10^5 \times 0.1 = C = 0.8 \times 10^4$

Max. pressure will be on inner radius

$$W = 2\pi C (r_1 - r_2)$$

$$= 2\pi \times 0.8 \times 10^4 (0.15 - 0.10)$$

$$W = 2513.27\,N$$

Total torque, $T_{total} = 2 \cdot \frac{\mu W}{2} (r_1 + r_2)$

$$= 0.3 \times 2513.27 (0.15 + 0.1)$$

$$T = 188.49\,NM$$

Power transmitted, $P = \frac{2\pi N T}{60}$

$$P = \frac{2\pi \times 2000 \times 188.49}{60}$$

$$P = 39.477\,KW$$

② The external radius of a friction plate of a single clutch having both sides as effective, is 150mm. The power transmitted is 20KW at a speed of 1000rpm. The maximum intensity of pressure at any point of contact surface is $0.8\times10^5 N/m^2$. If the coefficient of friction is 0.3, then find
(i) The internal radius of friction plate and
(ii) Axial thrust with which the friction surfaces are held together.

Given data:

$r_1 = 150mm = 0.15m$ $\qquad r_2 = ?$

$P = 20KW = 2000\ W$ $\qquad W = ?$

$N = 1000rpm$

$P_{max} = 0.8\times10^5 N/m^2$

$\mu = 0.3$

Assuming uniform wear theory: $P_{max}.r = constant$

P_{max} will be at r_2

$P_{max}.r_2 = c$

$$P = \frac{2\pi NT}{60} \Rightarrow 2000 = \frac{2\pi(1000)T}{60}$$

$T = 190.98\ N\text{-}m$

(i) Internal radius:

$$W = 2\pi \times c \times (r_1 - r_2) = 2\pi \times 0.8\times10^5 \times r_2(0.15 - r_2)$$

$$W = 502654\, r_2(0.15 - r_2) \rightarrow ①$$

Frictional torque due to both sides active surface

$$T = 2\left[\frac{\mu W}{2}(r_1 + r_2)\right]$$

$$= 0.3 \times 502654\, r_2 (0.15 - r_2) \times (0.15 + r_2)$$

$$= 150796\, r_2 (0.15^2 - r_2^2)$$

Torque due to power transmission = Frictional torque

$$\therefore\ 190.98 = 1507996\, r_2 (0.15^2 - r_2^2)$$

$$r_2^3 - 0.0225 r_2 + 0.001266 = 0$$

$$r_2 = 0.0976$$

$$\underline{\underline{r_2 = 97\ mm}}$$

(ii) Axial thrust (W)

~~Wce~~

$$(1) \Rightarrow W = 502654 \times 0.097 (0.15 - 0.0975)$$

$$\underline{\underline{W = 2584\ N}}$$

2572 also ✓

(3) The external and internal radii of a friction plate of a single plate clutch are 120mm & 60mm respectively. The total axial thrust with which friction surfaces are held together is 1500N. For uniform wear, find the maximum, minimum and average pressure on the contact surfaces.

Sol. Given $r_1 = 120\ mm = 0.12\ m$, $r_2 = 60\ mm = 0.06\ m$

$W = 1500N$, For uniform wear, $p \cdot r = c$

$\because$ pr = constant p will be max. when r is min &

p will be min when r is max.

$$\therefore\ P_{max} = \frac{c}{r_2}\ ; \quad P_{min} = \frac{c}{r_1}$$

Also $W = 2\pi \cdot c\, (r_1 - r_2)$

$$W = 2\pi P_{max} \cdot r_2 (r_1 - r_2)\ ; \quad W = 2\pi P_{min} r_1 (r_1 - r_2)$$

$$P_{max} = \frac{W}{2\pi r_2(r_1-r_2)} \; ; \; P_{min} = \frac{W}{2\pi r_1(r_1-r_2)}$$

$$P_{max} = \frac{1500}{2\pi(0.06)(0.12-0.06)} \; ; \; P_{min} = \frac{1500}{2\pi(0.12)(0.12-0.06)}$$

$$P_{max} = 66314\ N/m^2 \; ; \; P_{min} = 33157\ N/m^2$$

$$\text{Average Pressure},\ P_{avg} = \frac{\text{Total axial thrust}}{\text{Area of c.s of contact surface}} = \frac{W}{\pi(r_1^2-r_2^2)}$$

$$P_{avg} = \frac{1500}{\pi(0.12^2-0.06^2)}$$

$$P_{avg} = 44209\ N/m^2$$

Multiplate clutch:

(1) A multiplate clutch has six plates (friction rings) on the driving shaft & six plates on the driven shaft. The external radius of friction surface is 115 mm, where as the internal radius is 80 mm. Assuming uniform wear and coeff. of friction as 0.1, find the power transmitted at 2000 rpm. Axial intensity of pressure is not to exceed 0.16 N/mm².

Sol: Given,

$n_1 = 6$ $\qquad r_1 = 115\ mm = 0.115\ m$

$n_2 = 6$ $\qquad r_2 = 80\ mm = 0.08$

$n = n_1 + n_2 - 1$ $\qquad \mu = 0.1$

$n = 6+6-1$ $\qquad N = 2000\ rpm$

$n = 11$ $\qquad P_{max} = 0.16\ N/mm^2 = 0.16 \times 10^6\ N/m^2$

For uniform wear

$$T = n \cdot \mu W R_m, \quad \text{where} \quad R_m = \frac{r_1 + r_2}{2}$$

$P \times r = \text{constant}$

$$R_m = \frac{0.115 + 0.08}{2} = 0.0975 \text{ mm}$$

$p_{max}\, r_2 = C$

$0.16 \times 10^6 \times 0.08 = C$

$C = 128 \times 10^2$

$W = 2\pi C(r_1 - r_2) = 2\pi \times 128 \times 10^2 (0.115 - 0.08)$

$W = 2814.86\text{N}$

$\therefore T = 11 \times 0.1 \times 2814.86 \times (0.0975)$

$T = 301.893 \text{ N-m}$

$$\text{Power}, P = \frac{2\pi NT}{60} = \frac{2\pi (2000)(301.893)}{60}$$

$P = 63228 \text{ Watt}$

$P = 63.228 \text{KW}$

(2) A power of 60 KW is transmitted by a multiplate clutch at 1500rpm. Axial intensity of pressure is not to exceed 0.15 N/mm². The coeff of friction for the friction surface is 0.15. The external radius of friction surface is 120mm. Also the external radius is 1.25 times the internal radius. Find the number of plates needed to transmit the required power. Assume uniform wear.

Sol. $P = 60\text{KW} = 60 \times 10^3 \text{ W}$, $N = 1500 \text{rpm}$, $P_{max} = 0.15 \text{ N/mm}^2$

$\mu = 0.15$. $r_1 = 120\text{mm} = 0.12\text{m}$, $r_1 = 1.25 r_2$

$$r_2 = \frac{0.12}{1.25} \Rightarrow r_2 = 0.096 \text{ m}$$

For uniform wear

$$P \times r = \text{Constant}$$

$$\therefore P_{max} \times r_2 = C$$

$$C = 0.15 \times 10^6 \times 0.096 = 14400$$

$$W = 2\pi C (r_1 - r_2) = 2171.47 \text{N}$$

Power, $P = \dfrac{2\pi N T}{60}$; $\therefore T = \dfrac{60 \times P}{2\pi N} = \dfrac{60 \times 60 \times 10^3}{2\pi (1500)}$

torque, $T = 381.972$ N-m ✓

Also, $T = n \cdot \mu \cdot W \cdot R_m$

$R_m = \dfrac{r_1 + r_2}{2} = \dfrac{120+96}{2}$

$R_m = 108$ mm

$$381.972 = n \times 0.15 \times 2171.47 \times 0.108$$

$$n = 10.85 \text{ or}$$

$$n \simeq 11 \checkmark$$

No. of friction surfaces, $n = n_1 + n_2 - 1 = 11$

$$n_1 + n_2 = 11 + 1 = 12$$

Hence there will be total 12 plates, six plates for friction liners revolving with flywheel and 6 plates for disc revolving with driven shaft.

Cone Clutch Problem :-

(1) A cone clutch of cone angle 30° is used to transmit a power of 10KN at 800 rpm. The intensity of pressure b/w the contact surfaces is not to exceed 85 KN/m². The width of the conical friction surface is half of the mean radius. If coeff. of friction is 0.15, then find the dimensions of the contact surfaces. Assume uniform wear. Also find the axial load (or force req. to hold the clutch while transmitting the power.) What is the width of the friction surface.

Sol. Given;

Cone angle = 30° ; semi cone angle, $\alpha = 15^\circ$

power, $P = 10KW = 10\times10^3 W$; $N = 800$ rpm

$P_{max} = 85 \, KN/m^2 = 85\times10^3 \, N/m^2$, width, $b = \frac{1}{2} R_m$

$b = \frac{r_1+r_2}{4}$. $\mu = 0.15$

$P = \frac{2\pi NT}{60} \Rightarrow 10\times10^3 = \frac{2\pi(800)T}{60} \Rightarrow T = 119.366$ N-m

$b = \frac{r_1+r_2}{4}$ & $b = \frac{r_1-r_2}{\sin\alpha} \Rightarrow \frac{r_1-r_2}{\sin 15} = \frac{r_1+r_2}{4}$

$\boxed{r_1 = 1.138 r_2}$ $\quad p\times r = c$

$P_{max} \cdot r_2 = c \Rightarrow W = 2\pi c(r_1 - r_2)$

$T = \frac{1}{2}\frac{\mu W}{\sin\alpha}(r_1+r_2)$

$r_2 = 138$ mm ; $r_1 = 157$ mm

$W = 1400$ N & $b = 73.4$ mm

$\to 1317$N

② A cone clutch of semi cone angle 15° is used to transmit a power of 30KW at 800rpm. The mean frictional surface radius is 150mm. The normal intensity of pressure at the mean radius is not to exceed 0.15 N/mm². The coeff. of friction is 0.2. Assuming uniform wear theory determined.

(i) width of the contact surface

(ii) Axial load (or) force need to engage the clutch.

Sol. $\alpha = 15^\circ$; $P = 30KW = 30 \times 10^3 W$; $N = 800rpm$, $R_m = 150mm$

$p = 0.15 \times 10^6 N/m^2$ $\mu = 0.2$ $P = \frac{2\pi NT}{60}$;

$\therefore T = 358.\ N-m$

$T = \mu W_n R_m \Rightarrow 358.1 = 0.2 \times W_N \times 0.15$

$W_N = 11936 N$

W_N = Total load, normal to friction surface of the cone

$W_N = P_N \times 2\pi R_m \times b$

$11936 = 0.15 \times 10^6 \times 2\pi \times 0.15 \times b \Rightarrow \boxed{b = 84\ mm}$

$W = W_N \times \sin\alpha = 1193 \times \sin 15 = \boxed{3089\ N}$

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Centrifugal clutch :-

① A centrifugal clutch, has a driving member consist of a slider carrying four blocks which are kept from contact with the clutch by means of small spring until increase of centrifugal force overcomes the resistance of the springs and the power transmitted by friction b/w the shoes and case. Determine the necessary mass of each shoe if 22.5 KW power is to be transmitted at 750 rpm; with engagement begining at 75% of the scoriring speed. The inside diameter of the drum is 30 cm and the radial distance of the C.G of each shoe from the shaft is 12.5 cm. Assume $\mu = 0.25$

Sol, Given, $P = 22.5\,KW = 22.5 \times 10^3\,W$; $n = 4$

$N = 750\,rpm$; $x = 75\% = 0.75$

$D = 30cm = 0.3m$; $R = 0.15m$; $r = 12.5\,cm = 0.125m$

$\mu = 0.25$; $m = ?$

Centrifugal force $F_c = m\omega^2 r$ $\qquad F_s = m\omega_1^2 r$ (Spring force)

$F_c = m\left(\frac{2\pi \times 750}{60}\right)^2 \times 0.125$; $F_s = m\left(0.75 \times \frac{2\pi \times 750}{60}\right)^2 \times 0.125$

$F_c = \_\_\_\_\_ m$; $F_s = \_\_\_\_\_ m$

$R_N = F_c - F_s$

$T = \mu \underbrace{R_N}_{(F_c - F_s)} R \times n$ $\qquad P = \frac{2\pi NT}{60}$

$T = 286\,N\text{-}m$

$\therefore m = 5.65\,Kg$ → Mass of each block.