LECTURE NOTES

Unit I - Electrical Circuits & Measurements

Voltage (or Electrical Potential) :-

Potential difference in electrical terminology is known as voltage, generally measured between two points and its unit is the volt. It is expressed in terms of energy (W) per unit charge (Q) i.e

$$V = \frac{dW}{dQ} \quad \text{(volts)}$$

NOTE : If the work done in moving a charge of one coulomb between the two points is one joule, then the potential of the one point with reference to the second point is one volt.

Current :-
Current is defined as the rate of flow of electrons (charge) in a conductive or semiconductive material.

$$I = \frac{dq}{dt} \quad \text{(Amperes)}$$

NOTE : One ampere is equal to one coulomb per second.

Power :- The rate at which the work is done is power.

$$P = \frac{dW}{dt} = \frac{dw}{dq} \times \frac{dq}{dt}$$

$$= V \times I$$

$$P = VI \quad \text{(Watts)}$$

$P = VI = V\,V/R = V^2/R$

$P = VI = IRI = I^2R$

NOTE : One watt is the amount of power generated when one joule of energy is consumed in one second.

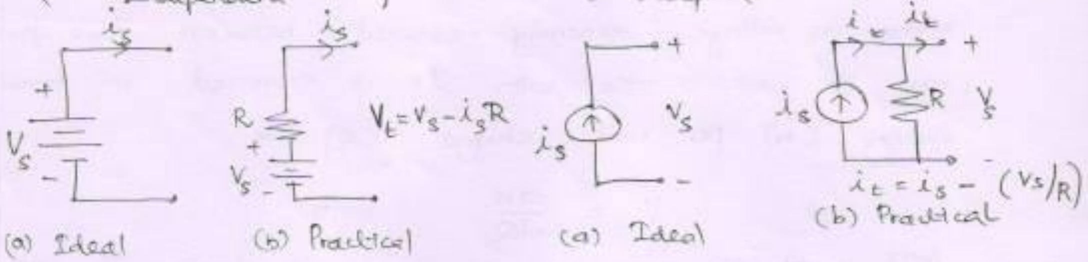
Energy :- Energy is the capacity for doing work.
(or)
The total amount of work done is known as Energy.

$$W = P \times t \quad \text{(Watts - hour)}$$

Active Elements :

An independent source which can deliver energy continuously is called an active element.

(i) Independent Voltage Source (ii) Independent Current Source



NOTE : The internal resistance of the practical voltage source is very small. The internal resistance of the practical current source is very high.

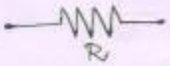
Passive Elements :

Passive elements are capable of receiving power.

Ex : Resistance, Inductance, Capacitance.

NOTE : Inductors & capacitors are capable of storing a finite amount of energy and return it later to an external element.

Resistance : Resistor is a dissipative element. Resistance is defined as the property of a substance which opposes the flow of electricity through it.



$$R = \rho \frac{l}{A} \quad (\Omega)$$

$l \rightarrow$ length of conductor ; $\rho \rightarrow$ Resistivity of material (constant)
$A \rightarrow$ Cross sectional area of the conductor.

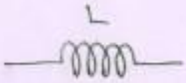
Note : Conductors offer very little resistance & hence radialy allow electricity to flow through them, whereas insulating materials offer such a high resistance that they allow practically no electricity to flow through them.

In pure metals R ↑s with an ↑ in temp.
R of carbon, electrolytes & insulating materials ↓s with ↑ in temp.
In alloys, R ↑s very slightly with ↑ in temp.

Inductance :

Inductance is a storage element which can store and deliver energy but its energy handling capacity is limited. Its unit is Henry (H).

It is like a coil wound on a magnetic core.



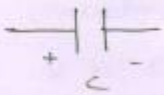
Emf induced in an inductance,

$$e = -L \frac{di}{dt}.$$

Capacitance :

Capacitance is a storage element which can store and deliver energy in electric field. Its unit is Farad (F).

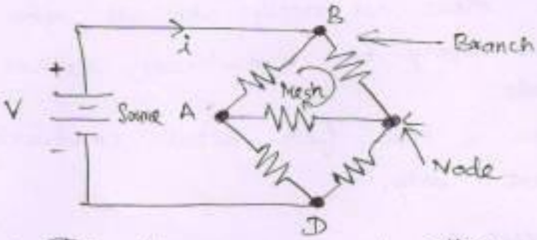
Any two metal plates between which an electric field can be maintained constitutes a capacitor.



$$C = \frac{Q}{V} \quad ; \quad i = C\frac{dv}{dt}$$

Electric Circuit :

Fig shows a typical electric circuit having a no of resistances connected together along with a voltage source.



(i) Network : The interconnection of either passive elements or the interconnections of active and passive elements constitute an electric n/w.

(ii) Node : A point where two or more than two elements are joined together is called a node.

(iii) Path : The movement through elements from one node to another without going through the same node twice is called as path.

(iv) Branch : An element or a no of elements connected between two nodes constitutes a branch.

(v) Loop : A closed path for the flow of current is called a loop.

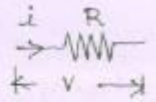
(vi) Mesh : A loop that does not contain any other loops within it is called a mesh.

NOTE :- For convenience, the nodes are usually labelled by letters.

In fig, there are 4 nodes : A, B, C & D
5 Branches : AB, BC, CD, DA & AC
2 Meshes : ABCA, ACDA
3 loops : ABCA, ACDA, ABCDA

OHM'S LAW :-

Ohm's law states that the current flowing through a conductor is directly proportional to the potential difference existing between the two ends of the conductor, provided the temperature remains constant, i.e $I \propto V$.



$\boxed{I \propto V}$

$I = V/R$; or $V = IR$; $R = V/I$

Limitations :

→ Ohm's law does not apply to all non-metallic conductors.
→ It does not apply to non-linear devices such as Zener diode.
→ Ohm's law is true for metal conductors at constant temperature only.

KIRCHOFF'S LAWS :

Kirchoff discovered two basic laws concerning n/ws, one commonly known as Kirchoff's current law (First law) whereas the second is called Kirchoff's voltage law (Second law)

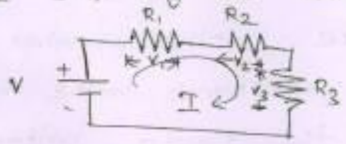
KCL : Kirchoff's current law states that the sum of the current flowing towards a node is equal to the sum of the current flowing away from that node.

i_1, i_2, i_5 are incoming ct's.
i_4, i_3 are outgoing ct's.
$\therefore\ i_1 + i_2 + i_5 = i_3 + i_4$

KVL : Kirchoff's voltage law states that the algebraic sum of the product of current and resistance of various branches of a closed mesh of a circuit plus the algebraic sum of the emfs in that closed mesh is equal to zero.

$$\Sigma IR + \Sigma E = 0$$

(or), In a closed mesh, The sum of the voltage rises is equal to the sum of the voltage drops.



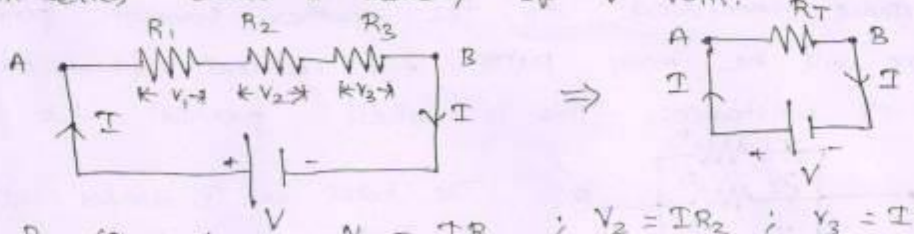
$$V = V_1 + V_2 + V_3$$

i.e $V = IR_1 + IR_2 + IR_3$

Resistance in Series & Voltage Division Technique :

The series ckt acts as a voltage divider. Since the same current flows each resistor, the voltage drops are proportional to the values of resistors.

Let three resistances R_1, R_2 & R_3 be connected in series across a battery of V volts.



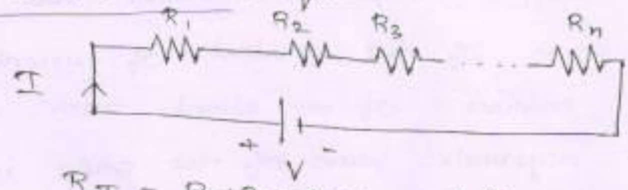
By Ohm's law, $V_1 = IR_1$; $V_2 = IR_2$; $V_3 = IR_3$

Also,
$$V = V_1 + V_2 + V_3$$
$$= IR_1 + IR_2 + IR_3$$
$$V = I(R_1 + R_2 + R_3)$$
(or)
$$\frac{V}{I} = R_1 + R_2 + R_3$$
$$\boxed{R_T = R_1 + R_2 + R_3} \longrightarrow (1)$$

Also $\frac{1}{G_T} = \frac{1}{G_1} + \frac{1}{G_2} + \frac{1}{G_3}$ (interms of conductance)

When a number of resistances are connected in series, the equivalent resistance is the sum of all the individual resistances.

Voltage Division Technique :



$$R_T = R_1 + R_2 + R_3 + \dots + R_n$$

$$I = V/R_T$$

$$V_1 = IR_1 \;;\; V_2 = IR_2 \;;\; V_3 = IR_3 \;;\; \dots \; V_n = IR_n$$

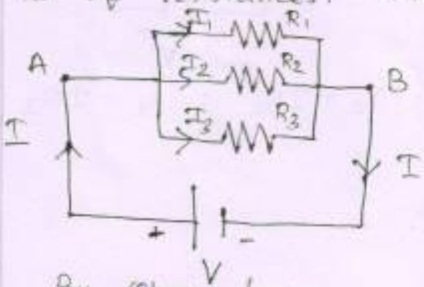
$$V_1 = \frac{V}{R_T} R_1 \;;\; V_2 = \frac{V}{R_T} R_2 \;;\; V_3 = \frac{V}{R_T} R_3 \dots V_n = \frac{V}{R_T} R_n$$

∴ Voltage across any resistance in the series ckt is equal to the ratio of that resistance value to the total resistance, multiplied by the source voltage, i.e

$$V_x = \frac{R_x}{R_T} V_s$$

Resistances in Parallel & Current Division Technique :-

If one end of ~~the~~ each resistance, connected to one common point and the other end of each resistance connected to the another common point, there will be many paths for current flow as the no of resistances. This is called parallel ckt.



The total ct I divides into 3 parts.

Voltage across each resistance is same.

By Ohm's law,

$$I_1 = V/R_1 \;;\; I_2 = V/R_2 \;;\; I_3 = V/R_3$$

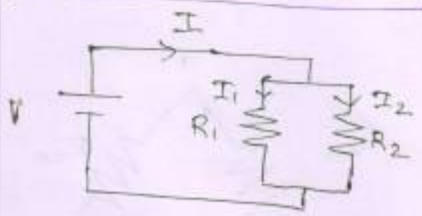
Also

$$I = I_1 + I_2 + I_3$$
$$= V/R_1 + V/R_2 + V/R_3$$
$$= V(1/R_1 + 1/R_2 + 1/R_3)$$
$$I/V = 1/R_1 + 1/R_2 + 1/R_3$$

$$\Rightarrow \boxed{1/R_T = 1/R_1 + 1/R_2 + 1/R_3}$$ Also $G_T = G_1 + G_2 + G_3$

Hence when a no of resistances are connected in parallel, the reciprocal of the total resistance is equal to the sum of reciprocals of individual resistances.

Current Division Technique :



In a parallel ckt, the ct divides in all branches. Thus, a parallel ckt acts as a ct divider.

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$= \frac{R_2+R_1}{R_1R_2} \Rightarrow R_T = \frac{R_1R_2}{R_1+R_2};$$

Branch cts, I_1, I_2 :-

$$V = I R_T = I\left(\frac{R_1R_2}{R_1+R_2}\right)$$

$$I_1 = V/R_1 = \frac{I\left(\frac{R_1R_2}{R_1+R_2}\right)}{R_1} = I\left(\frac{R_2}{R_1+R_2}\right)$$

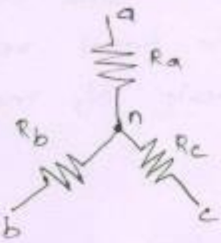
$$I_2 = V/R_2 = \frac{I\left(\frac{R_1R_2}{R_1+R_2}\right)}{R_2} = I\left(\frac{R_1}{R_1+R_2}\right)$$

Hence in a parallel ckt, of two resistances, the ct in one resistor is the total ct × opposite R / Total Resistance

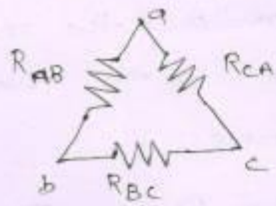
Difference b/w Series & Parallel ckts

Series ckt	Parallel ckt
1. $R_T = R_1+R_2+R_3+\dots+R_n$	$1/R_T = 1/R_1 + 1/R_2 + \dots + \frac{1}{R_n}$
2. Ct flowing through all the resistances will be same & equal to the total ct.	Ct flowing through each resistance is different.
3. The voltage is divided across each resistance according to the value of resistance	The voltage across each resistance is same which will be equal to the input voltage.
4. $E = E_1+E_2+E_3+\dots+E_n$	5. $I = I_1+I_2+\dots+I_n$

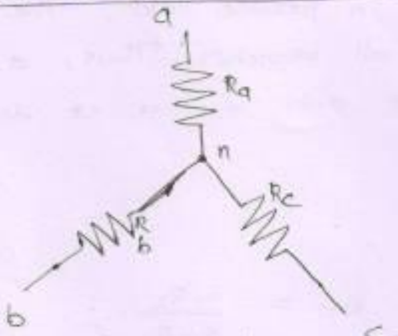
Star Connected N/w



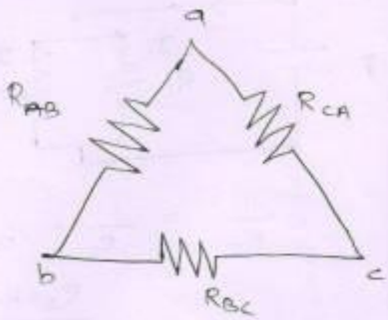
Delta Connected N/w



Star to Delta Conversion :-



$\Rightarrow$

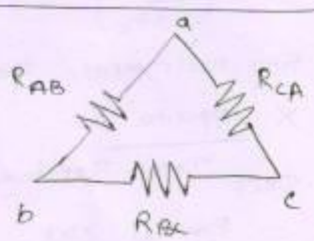


$$R_{AB} = \frac{R_a R_b + R_b R_c + R_c R_a}{R_c}$$

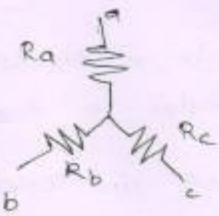
$$R_{BC} = \frac{R_a R_b + R_b R_c + R_c R_a}{R_a}$$

$$R_{CA} = \frac{R_a R_b + R_b R_c + R_c R_a}{R_b}$$

Delta to Star Conversion :-



$\Rightarrow$



$$R_a = \frac{R_{AB} \; R_{CA}}{R_{AB} + R_{BC} + R_{CA}}$$

$$R_b = \frac{R_{AB} \; R_{BC}}{R_{AB} + R_{BC} + R_{CA}}$$

$$R_c = \frac{R_{BC} \; R_{CA}}{R_{AB} + R_{BC} + R_{CA}}$$

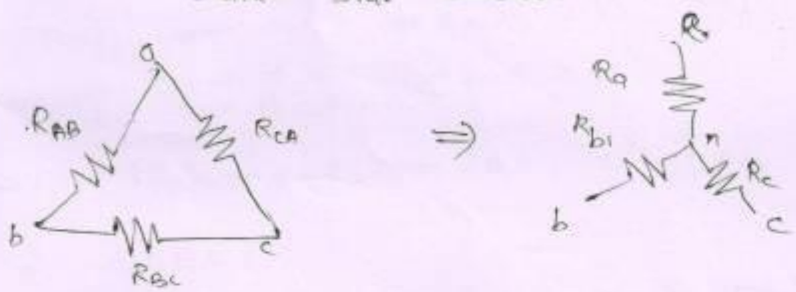
Star - Delta Conversion

$$R_{AB} = \frac{R_a R_b + R_b R_c + R_c R_a}{R_c}$$

$$R_{BC} = \frac{R_a R_b + R_b R_c + R_c R_a}{R_a}$$

$$R_{CA} = \frac{R_a R_b + R_b R_c + R_c R_a}{R_b}$$

Delta - Star Conversion



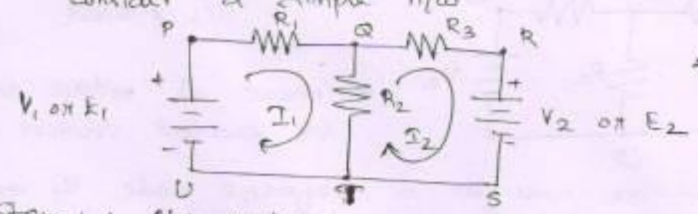
$$R_a = \frac{R_{AB} \times R_{CA}}{R_{AB} + R_{BC} + R_{CA}}$$

$$R_b = \frac{R_{AB}\, R_{BC}}{R_{AB} + R_{BC} + R_{CA}}$$

$$R_c = \frac{R_{BC}\, R_{CA}}{R_{AB} + R_{BC} + R_{CA}}$$

Mesh Current Analysis

Consider a simple n/ω



* To determine loop (or) mesh ckts
* KVL is used

Step 1 : Ckt indicates that there are 2 loops PQTUP & QRSTQ in the n/ω.

Let I_1 & I_2 be the loop currents.

Considering loop PQTUP alone, I_1 is passing through R_1, and $(I_1 - I_2)$ is passing through R_2.

By applying KVL,

$$V_1 = I_1 R_1 + R_2 (I_1 - I_2) \longrightarrow (1)$$

Similarly, in loop QRSTQ, I_2 is passing through R_3, and $(I_2 - I_1)$ is passing through R_2.

KVL $\Rightarrow$ $-V_2 = R_2 (I_2 - I_1) + I_2 R_3 \longrightarrow (2)$

Mesh currents I_1 & I_2 can be found out by solving eqns 1 & 2.

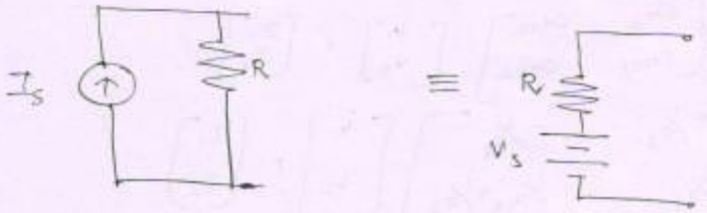
The branch currents can be easily found out by using mesh currents I_1 & I_2.

Source Transformation :

$I_s = \frac{V_s}{R}$

$R = R$

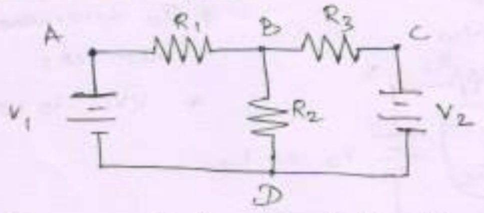
Voltage Source   Current source



$V_s = I_s \times R$

$R = R$

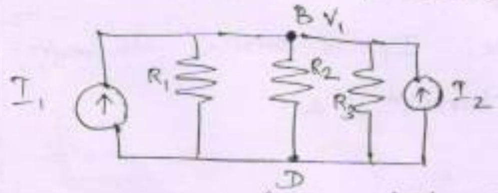
Nodal Analysis



* To find node voltages
* KCL is used
* Convert all voltage sources to current sources.

Step 1: Select one node as a reference node & assign each of the remaining nodes its own unknown potential. (D)

Step 2: Let B be the principal node & V_1 be the potential at 'B'.

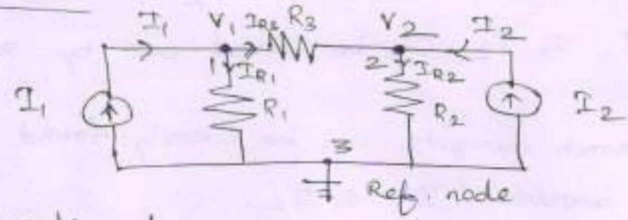


Step 3: Apply KCL at node B

$$I_1 + I_2 = \frac{V_1}{R_1} + \frac{V_1}{R_2} + V_1/R_3$$

By solving this eqn, we can find V_1.

Ex ckt:



At node 1,

$$\text{KCL} \Rightarrow \quad I_1 = \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_3} \rightarrow 1$$

At node 2,

$$I_2 = \frac{V_2 - V_1}{R_3} + \frac{V_2}{R_2} \rightarrow 2$$

By solving eqns 1, 2 we can find V_1, V_2.

Alter method

$$G V = I$$

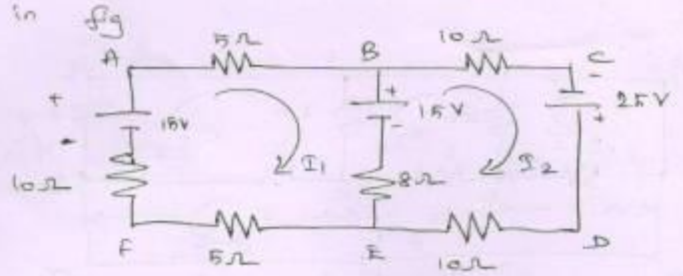
n=2 nodes

$$\begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} 1/R_1 + 1/R_3 & -1/R_3 \\ -1/R_3 & 1/R_2 + 1/R_3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

By using Cramer's rule we can find V_1 & V_2.

1. Find the ct in the 8Ω resistor in the ckt shown in fig



Apply KVL to mesh ABEFA,

$$-10I_1 + \cancel{15} - 5I_1 - \cancel{15} - 8(I_1 - I_2) - 5I_1 = 0 \rightarrow 1$$

$$-28I_1 + 8I_2 = 0$$

$$I_2 = 3.5 I_1 \rightarrow A$$

Apply KVL to mesh BCDEB,

$$-10I_2 + 25 - 10I_2 - 8(I_2 - I_1) + 15 = 0 \rightarrow 2$$

$$-28I_2 + 8I_1 + 40 = 0$$

$$28I_2 - 8I_1 = 40$$

$$7I_2 - 2I_1 = 10 \rightarrow B$$

Apply A in B

$$7(3.5 I_1) - 2I_1 = 10$$

$$22.5 I_1 = 10$$

$$I_1 = 0.44 \text{ A}$$

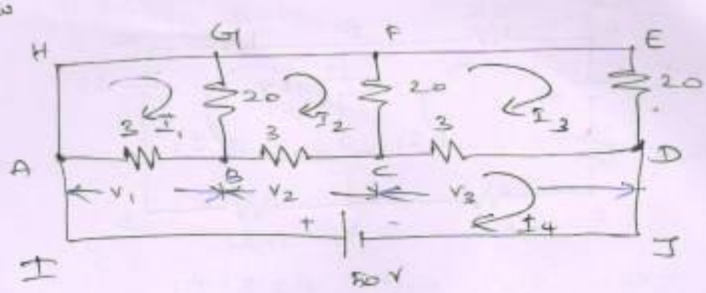
$$I_2 = 1.55 \text{ A}$$

∴ Current through 8Ω Resistor, $= I_2 - I_1$

$$= 1.11 \text{ A}$$

Hence ct through 8Ω resistance is 1.1 A & flows from E to B.

7 Determine the voltages across the 3Ω resistors in the n/w



ABGHA,

$$-3(I_1 - I_4) - 20(I_1 - I_2) = 0$$

$$-23I_1 + 20I_2 + 3I_4 = 0 \rightarrow 1$$

GFCBG

$$-20(I_2 - I_3) - 3(I_2 - I_4) - 20(I_2 - I_1) = 0$$

$$20I_1 - 43I_2 + 20I_3 + 3I_4 = 0 \rightarrow 2$$

FEDCF,

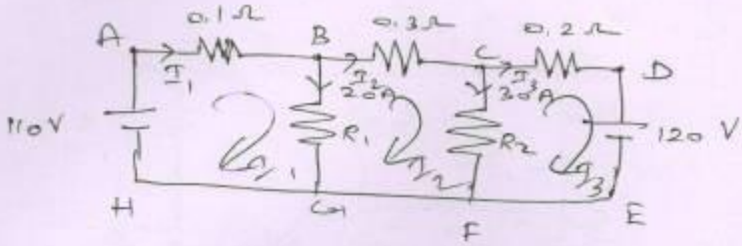
$$-20I_3 - 3(I_3 - I_4) - 20(I_2 - I_2) = 0$$

$$20I_2 - 43I_3 + 3I_4 = 0 \rightarrow 3$$

ABCDJIA,

$$-3(I_4 - I_1) - 3(I_4 - I_2) - 3(I_4 - I_3) + 50 = 0$$

$$3I_1 + 3I_2 + 3I_3 - 9I_4 + 50 = 0 \rightarrow 4$$



Apply KCL at nodes B & C

$I_1 = 20 + I_2 \longrightarrow 1$

$I_2 = I_3 + 30 \longrightarrow 2$

Apply KVL ABGHA

$110V - 0.1 I_1 - 20 R_1 = 0 \longrightarrow 3$

$2 + 0.1 I_2 + 20 R_1 = 0$

KVL → BCFGB

$0.3 I_2 + 30 R_2 - 20 R_1 = 0 \longrightarrow 4$

CDEFC.

$0.2 I_3 - 120 - 30 R_2 = 0 \longrightarrow 5$

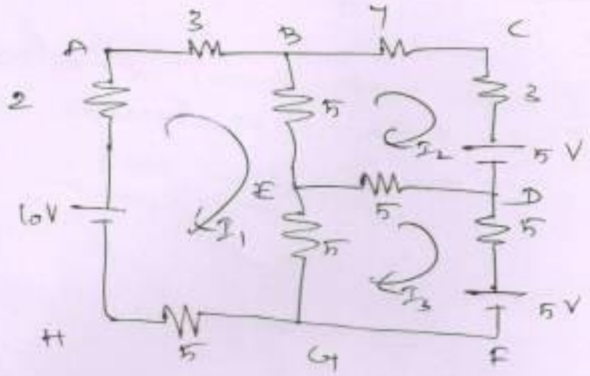
$+0.1 I_1 + \cancel{20 R_1} = 110$

$0.3 I_2 + 30 R_2 - \cancel{20 R_1} = 0$

$-$

$0.1 I_1 + 0.3 I_2 + 30 R_2 = 110$

Determine the power o/p of each voltage source, using Kirchoff's laws, for the n/w.



ABGHA

$$10 - 2I_1 - 3I_1 - 5(I_1 - I_2) - 5(I_1 - I_3) - 5I_1 = 0$$

$$-20I_1 + 5I_2 + 5I_3 = -10 \rightarrow 1$$

$$-4I_1 + I_2 + I_3 = -2 \rightarrow (1)$$

BCDEB

$$-7I_2 - 3I_2 - 5 - 5(I_2 - I_3) - 5(I_2 - I_1) = 0$$

$$5I_1 - 20I_2 + 5I_3 = 5 \rightarrow 2$$

$$I_1 - 4I_2 + I_3 = 1 \rightarrow (2)$$

$$-5(I_3 - I_2) - 5I_3 - 5 - 5(I_3 - I_1) = 0$$

$$-15I_3 + 5I_2 + 5I_1 = 5 \rightarrow 3$$

$$-3I_3 + I_2 + I_1 = 1 \rightarrow (3)$$

$I_1 = 0.37A$; $I_2 = -0.23A$,

$I_3 = -0.29A$,,

P_{out} : 10 V : $10 \times 0.37 = 3.7$ W

5 V : $5 \times 0.23 = 1.15$ W

5 V $5 \times 0.29 = 1.45$ W,,

A copper wire is 500 m long has a diameter of 1 mm. Find its resistance if the resistivity of copper is $1.73 \times 10^{-8}\ \Omega m$

$$R = \rho \frac{l}{A} = \frac{1.73 \times 10^{-8} \times 500}{\pi \left(\frac{1 \times 10^{-3}}{2}\right)^2} = 11.0135\ \Omega$$

2. What will be the current drawn by a lamp rated at 250 V, 40 watts connected to a 230 V supply?

Rated Power = 40 watts

Rated Voltage = 250 V

$$P = \frac{V^2}{R}$$

$$40 = \frac{250^2}{R}$$

$$R = 1562.5\ \Omega$$

Current drawn from 230 V supply $= V/R = \frac{230}{1562.5} = 0.1472$ Amp.

3. Twenty lamps each of 60 watts are used each for 4 hours per day in a building. Calculate (i) the current drawn when all the lamps are working and (ii) the monthly electricity at 55 paise per unit. Assume a supply of 240 V.

Soln :

Current drawn by one lamp $= P/V = \frac{60}{240} = 0.25\,A$

Total ck drawn by 20 lamps $= 20 \times 0.25 = 5$ A

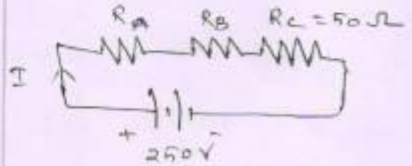
Energy consumed in a month $= 30 \times 4 \times 20 \times 60$ Wh

$$= \frac{30 \times 4 \times 20 \times 60}{1000}\ \text{kWh}$$

$= 144$ units

Monthly electric charge $= 144 \times 0.55 =$ Rs 79.20

4.



1

AC Circuits – Waveforms, RMS value – Power Power factor – 1φ & 3φ balanced ckts

Generation of Alternative Emf :

Intro When the current flowing in the ckt varies in magnitude as well as direction periodically, it is called alternating ct.

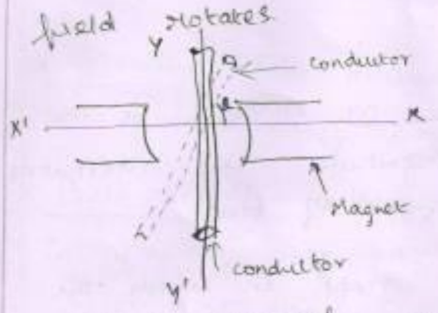
An alternating ct or voltage is one, which periodically passes through a definite cycle, consisting of two half cycles – during one of which the ct or voltage varies in one direction and during the other half cycle, in the opposite direction.

The ckts in which alternating cts flow are called AC ckts.

Generation : When a conductor is rotated in a magnetic field, an alternating emf is generated in the conductor.

An alternating emf will also be generated by changing the magnetic field with in the stationary coil.

The Emf generated will depend upon the strength of the magnetic field, the no of turns in the coil and the speed at which the coil or the magnetic field rotates.



$\theta = \omega t$

$v = \omega r$ m/s (r → radius of the path)

$v \sin\theta = v \sin \omega t$

Flux cut per second $= B l v \sin\theta$

l → length of coil ; B → flux density

Emf generated at time t, $= B l v \sin \omega t$

" in the coil side

" in the coil at time t $= 2 B l v \sin \omega t$

Max. flux linking the coil, $\phi = B \times 2 l r$

$\therefore$ Emf $= \phi \omega \sin \omega t$

$E_{max} = \phi \omega$

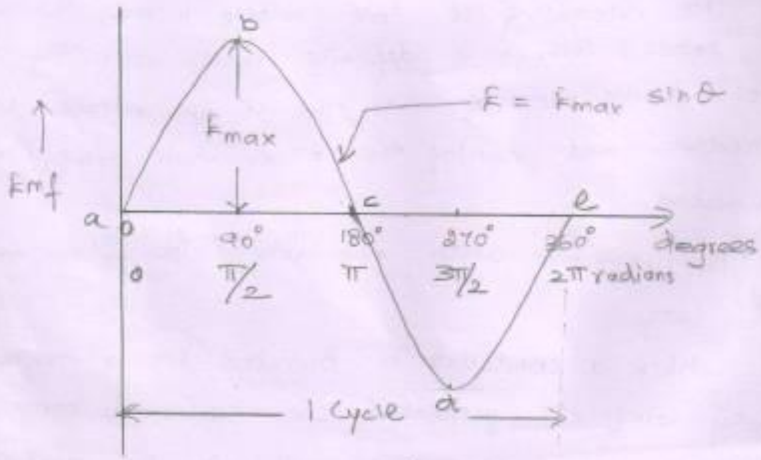
$E = E_{max} \sin \omega t$

$$i = I_{max} \sin \omega t$$

$$\omega = 2\pi f$$

$T = 1/f$ is called the periodic time & it is ~~g~~ the time taken to complete one cycle.

Sinusoidal Wave of Emf :



The trace abcde of the graph completes one cycle and consists of two alternations, one positive and other negative.

Such a wave will complete a certain no of cycles in one second, which is called the frequency of the wave, expressed in Hertz (Hz)

Terminology :

An alternating voltage or current is one, which changes continuously in magnitude and alternates in direction at regular interval of time.

Wave form : A wave form is a graph in which the instantaneous value of any quantity is plotted against time.

Alternating Waveform : This is a wave which reverse its direction at regularly recurring intervals.

Periodic Waveform : Periodic waveform is one which repeats itself after definite time intervals.

Alternating waveform in which sine law is followed is known as sinusoidal waveform.

Cycle : A complete set of +ve and -ve values of an alternating voltage or current.

A cycle of ac wave is normally specified in terms of angular measure spread over 360° or 2π radians.

Frequency : The no of cycles passed through in one second is called the frequency of the wave.

It is denoted by f & is expressed in Hz or c/s.

$$f = \frac{PN}{120} \quad \text{(in alternator)}$$

Periodic time :

The time taken by an alternating voltage or current to complete one cycle is termed its periodic time.

$$\text{Time period } T = 1/f \quad \text{(sec)}$$

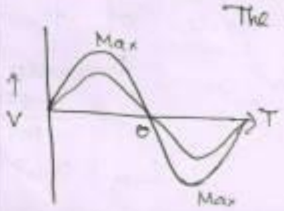
Angular velocity : The angular distance covered per second is defined as angular velocity.

$$\omega = 2\pi f \quad \text{(rad/sec)}$$

Amplitude : The maximum positive or negative value of an alternating quantity is called the amplitude.

Phase : Two alternating wave shapes are said to be in phase when they reach their maximum and zero values at the same time.

The term 'phase difference' is used to compare the phases of two w/f s.



The phase at any point on a gn wave is the time that has elapsed since the quantity has last passed through zero point of reference and passed positively.

4

Root Mean Square Value :

The RMS or effective value of ac is defined as that value of direct current which will do the same amount of work in the same time or would produce the same heating effect as when the alternating ct is applied.

By Graphical method,

$$I_{rms} = \sqrt{\text{Mean value of } (i)^2} \qquad \left(\sqrt{\frac{\text{Area under the square curve for one cycle}}{\text{period}}}\right)$$

$$E_{rms} = \sqrt{\text{Mean value of } (E)^2}$$

By analytical method,

$$W = \int_0^T i^2 R \, dt \qquad \begin{aligned} W &= P \times t \\ &= i^2 R \, dt \end{aligned}$$

$$V_{rms} = \left[\frac{1}{T}\int_0^T v^2 \, dt\right]^{1/2}$$

SINE wave

RMS value of voltage for sine sine wave $= \dfrac{\text{max. value of } V}{\sqrt{2}}$

" current " $= \dfrac{\text{max. value of } I}{\sqrt{2}}$

(or)

$$I_{rms} = 0.707 \; I_{max} \quad (I_m/\sqrt{2})$$

$$E_{rms} = 0.707 \; E_{max} \quad (V_m/\sqrt{2})$$

Average Value :-

of alternating wave is the arithmetic mean of the ordinates at equal intervals over a half cycle of that wave.

$$I_{avg} = \frac{i_1 + i_2 + \dots + i_n}{n} \qquad = \frac{\text{Area under one complete cycle}}{\text{Period}}$$

$$I_{avg} = \frac{1}{T}\int_0^T i \, dt \qquad ; \; I_{avg} = 0.637 \, I_m$$

$$V_{avg} = 0.637 \, V_m$$

Form factor, Peak factor :-

The relationship b/w avg, RMS & max. values can be expressed by these two factors.

$$\text{Form factor} = \frac{\text{RMS value}}{\text{Avg value}}$$

$$k_f = \frac{0.707\, I_m}{0.637\, I_m} = 1.11$$

$$\text{Peak factor} = \frac{\text{Peak value}}{\text{RMS value}}$$

$$k_p = \frac{I_m}{I_m/\sqrt{2}} = \sqrt{2} = 1.414$$

RMS Value of Sine wave

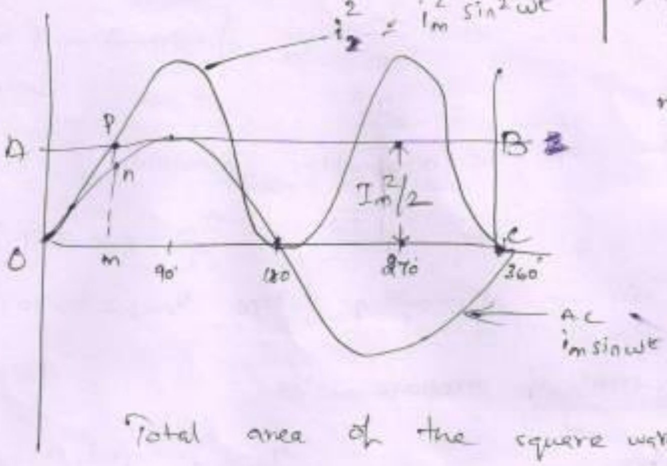
(i) Graphical Method :

$i = I_m \sin \omega t$

$= I_m \sin \theta \quad (\omega t = \theta)$

- → Draw AC waveform (i)
- → Draw the w/f (i^2)
- → Draw AB line ∥ el to OC, height $= I_m^2/2$
- → Area of the square wave = Area under OABC

$mp = (mn)^2$



Total area of the square wave is equal to the area of the ~~horizontal axis~~ ~~of~~ rectangle OABC

Thus mean value of $i^2 \times 2\pi = \frac{i_{max}^2}{2} \times 2\pi$

(or) mean value of $i^2 = \frac{i_{max}^2}{2}$

Hence, rms value of current of sine wave,

$$I = \sqrt{\text{mean value of } (i)^2}$$

$$= \sqrt{\frac{I_{max}^2}{2}} = \frac{I_{max}}{\sqrt{2}} = 0.707\, I_{max}$$

(or)

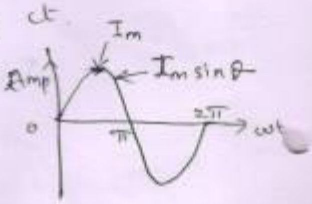
$$I = \frac{I_{max}}{\sqrt{2}} \; , \quad V = \frac{V_{max}}{\sqrt{2}}$$

Analytical Method :

Let $i = I_m \sin \omega t$ be the alternating ct

$i = I_m \sin \theta \quad (\omega t = \theta)$

$i^2 = I_m^2 \sin^2\theta$



Mean square of AC $= \frac{1}{2\pi} \int_0^{2\pi} i^2 \, d\theta$

$$= \frac{1}{2\pi} \int_0^{2\pi} I_m^2 \sin^2\theta \, d\theta$$

$$= \frac{I_m^2}{2\pi} \int_0^{2\pi} \sin^2\theta \, d\theta = \frac{I_m^2}{2\pi} \int_0^{2\pi} \left(\frac{1-\cos 2\theta}{2}\right) d\theta$$

$$= \frac{I_m^2}{2\pi} \cdot \frac{1}{2} \int_0^{2\pi} (d\theta - \cos 2\theta)\, d\theta$$

$$= \frac{I_m^2}{2\pi} \cdot \frac{1}{2} \left[\theta - \frac{\sin 2\theta}{2}\right]_0^{2\pi}$$

$$= \frac{I_m^2}{2\pi} \cdot \frac{1}{2} \left[(2\pi - 0) - (0 - 0)\right]$$

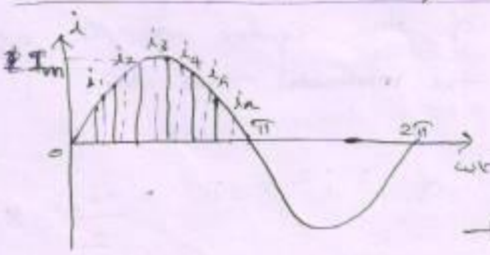
$$= \frac{I_m^2}{2}$$

RMS value of the alternating sinusoidal ct is

$$I_{rms} = \sqrt{\frac{I_m^2}{2}} = \frac{I_m}{\sqrt{2}} = 0.707\, I_m.$$

Similarly for an alternating voltage, $V_{rms} = 0.707\, V_m$.

Determination of average value :



$$I_{avg} = \frac{i_1 + i_2 + \dots + i_n}{n}$$

Avg value can be easily obtained by first finding the avg value for a small interval of time and then integrating over the curve. i.e $I_{av} = \frac{1}{T} \int_0^T i \, dt$

This is nothing but the ratio of the area under the curve over one complete cycle to the base.

Avg value for Symmetrical & unsymmetrical wave :

In the case of symmetrical ac or voltage wave there exists two exactly similar half cycles whether sinusoidal or non sinusoidal.

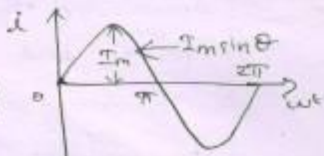
In this case, the avg value over a complete cycle is zero. Hence for symmetrical waves the avg value is taken for only one half cycle.

In the case of unsymmetrical waves, the avg values are always taken over the whole cycle.

Analytical Method to obtain the avg value for sinusoidal current :-

Let $i = I_m \sin\theta$ be the ac w/f.

Since this is a symmetrical wave, it has two equal half cycles, namely +ve & -ve halves.



Considering one half cycle for this symmetrical wave the avg value is obtained by,

$$I_{avg} = \frac{1}{\pi}\int_0^{\pi} i \, d\theta$$

$$= \frac{1}{\pi}\int_0^{\pi} I_m \sin\theta \, d\theta$$

$$= \frac{I_m}{\pi}\left[-\cos\theta\right]_0^{\pi}$$

$$= \frac{I_m}{\pi}\left[-(-1)+1\right]$$

$$I_{avg} = \frac{2I_m}{\pi} = 0.637\, I_m$$

For a sinusoidal voltage wave, $V_{avg} = 0.637\, V_m$.

Power in AC Circuits :-

Real Power (Active Power) :

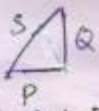
The actual power consumed in an ac ckt is called real power. If E and I are rms values of voltage and current respectively and θ is the phase angle between E & I then

$$P = EI \cos\phi \quad (\text{Watts})$$

The avg power drawn by a ckt is found out by multiplying the rms values of voltage & ct by cos φ, commonly termed as power factor. The avg power is also called as active power taken by the ckt and is measured in Watts

Apparent Power :

The product of rms values of voltage and current in ac ckts is normally greater than the active power drawn by the ckt. Such a product is termed as apparent power and is measured in volt-ampere (VA)



Power Angle Diagram

Apparent power, $S = VI$ (VA)

Complex power, $S = P + jQ$

Reactive Power

In ac ckts, current lags or leads the applied voltage by an angle φ. As such current can be resolved into active and reactive components. The reactive component of current is equal to $I \sin\phi$.

Power drawn by the ckt due to reactive component of current is called reactive power.

$$\text{Reactive power} = VI \sin\phi \quad (\text{VAR})$$

Power Factor :

Ratio of active power to apparent power is called the power factor of the ckt.

$$\text{Power factor} = \frac{VI\cos\phi}{VI} = \cos\phi$$

Hence, in ckts whose waveforms follow the sine law, power factor is the cosine of the angle between the applied voltage & the resultant current flowing in the ckt.

Power factor is normally called lagging when the ct lags the applied voltage and leading when the ct leads the voltage.

Power factor for ac ckts, can be written as

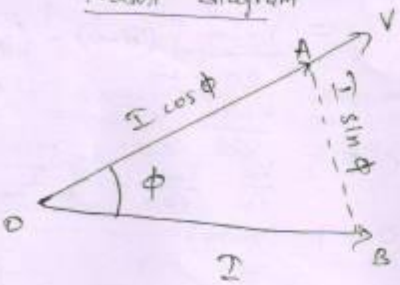
$$P.f = \cos\phi = \frac{V_R}{V} = \frac{IR}{IZ}$$

$$\cos\phi = \frac{R}{Z} = \frac{\text{Resistance}}{\text{Impedance}}$$

The p.f for an inductive ckt is always lagging whereas in capacitive ckt always leading.

Active & Reactive Power :

Phasor diagram



I is lagging behind V.

I can be resolved into two components.

(i) $OA = I\cos\phi$, in phase with the voltage termed as active or power component of ct. The power contributed by it is equal to $VI\cos\phi$.

(ii) $AB = I\sin\phi$, in quadrature with the voltage & called reactive or wattless component of ct.

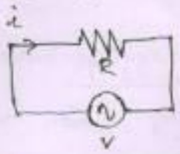
Hence, active component of ct $= I\cos\phi$

Reactive " " " $= I\sin\phi$

Analysis of AC Circuit :

The response of electric ckts to ac can be studied by passing an ac through R, L & C.

Pure Resistive Ckt : Resistor of value R ohms is connected across an alternating V source



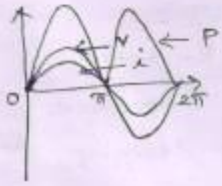
$v = V_m \sin \omega t$

By Ohm's law $v = iR$

$i = v/R = \frac{V_m \sin \omega t}{R} = I_m \sin \omega t$ $\left(\because \frac{V_m}{R} = I_m\right)$

Phasor Representation :

In pure resistive ckt, there is no phase difference b/w the voltage applied & the resulting ct, i.e phase angle $\phi = 0$



Impedance $Z = \frac{V_m}{I_m} = \frac{V_m}{V_m/R} = R$

Avg Power $P = vi = V_m \sin \omega t \times I_m \sin \omega t$

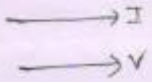
$= \frac{1}{\pi} \int_0^{\pi} V_m I_m \sin^2 \omega t \, d(\omega t)$

$= \frac{V_m I_m}{\pi} \int_0^{\pi} \left(\frac{1 - \cos 2\theta}{2}\right) d\theta$

$= \frac{V_m I_m}{2\pi} \left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}$

$= \frac{V_m I_m}{2\pi} \left[(\pi - 0) - (0 - 0)\right]$

$= \frac{V_m I_m}{2\pi} \times \pi = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}}$

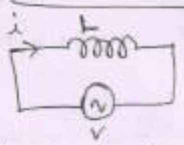


$\phi = 0$

Avg power, $P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} = V_{rms} I_{rms}$ Watts

Power factor : $\cos \phi = \cos 0 = 1$ (unity)

Pure Inductive ckt :



In this ckt, an alternating voltage is applied across a pure inductor of self inductance L Henry.

Let $v = V_m \sin \omega t$; $\rightarrow 1$

We know that self induced emf always opposes the applied V.

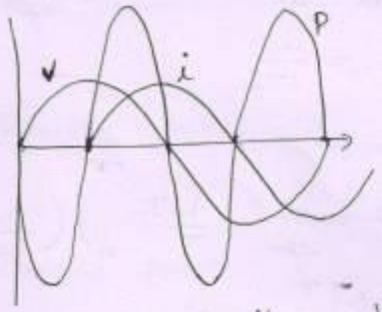
$v = L \, di/dt$;

$i = \frac{1}{L} \int v \, dt = \frac{1}{L} \int V_m \sin \omega t \, dt = \frac{V_m}{L}\left(-\frac{\cos \omega t}{\omega}\right)$

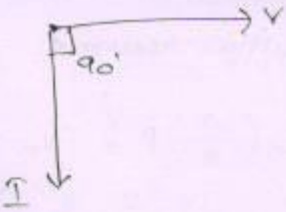
$i = -\frac{V_m}{\omega L} \cos \omega t = I_m \sin(\omega t - \pi/2) \rightarrow 2$

Comparing eqns 1 and 2, the ct through an inductor lags the applied voltage by 90°.

Waveform Representation :



Phasor Representation



Impedance $Z = \frac{V_m}{I_m} = \frac{V_m}{\frac{V_m}{\omega L}} = \omega L$

$Z = \omega L$ is called inductive reactance & is denoted by X_L.

$$X_L = \omega L = 2\pi f L$$

Power $p = vi = (V_m \sin\theta)(I_m \sin(\theta - \pi/2))$

$$p = (V_m \sin\theta)(-I_m \cos\theta)$$

$$= \frac{-1}{\pi}\int_0^{\pi} (V_m I_m \sin\theta \cos\theta)\, d\theta$$

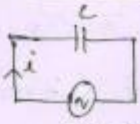
$$= \frac{V_m I_m}{\pi}\int_0^{\pi} \left(\frac{\sin 2\theta}{2}\right) d\theta$$

$$P = \frac{V_m I_m}{2\pi}\left(\frac{\cos 2\theta}{2}\right)_0^{\pi} = \frac{V_m I_m}{2\pi}(1-1) = 0.$$

Thus, a pure inductor does not consume any real power.

Power factor $\cos\phi = \cos 90° = 0$ (lagging)

Pure Capacitive Ckt



$V = V_m \sin\omega t$

$v = V_m \sin \omega t$; $\rightarrow$ (1)

The characteristic eqn of a capacitor is $v = \frac{1}{C}\int i\, dt$

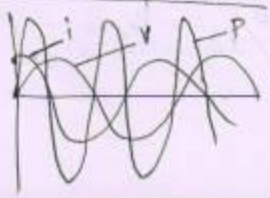
$$i = C\frac{dv}{dt} = C\frac{d}{dt}(V_m \sin\omega t) = \omega C V_m \cos\omega t$$

$$i = I_m \cos\omega t \quad (\because I_m = \omega C V_m)$$

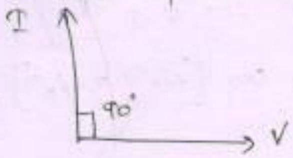
$$i = I_m \sin(\omega t + 90°) \rightarrow (2)$$

Comparing eqns 1 & 2, there is a phase difference of 90° between the voltage and the current in a pure capacitor.

Wave form Representation



Phasor Representation



Impedance $Z = \frac{I_m}{V_m} = \frac{V_m}{\omega C V_m} = \frac{1}{\omega C} = X_C$

Capacitive reactance $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

Power $P = V_m I_m \sin\theta \cos\theta$

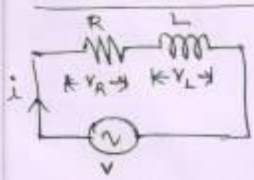
$= \frac{1}{\pi} \int_0^{\pi} V_m I_m \sin\theta \cos\theta \, d\theta = \frac{V_m I_m}{\pi} \int_0^{\pi} \left(\frac{\sin 2\theta}{2}\right) d\theta$

$= \frac{V_m I_m}{2\pi} \left[-\frac{\cos 2\theta}{2} \right]_0^{\pi} = 0$

Thus the pure capacitor does not consume any real power.

Power factor : $\cos\phi = \cos 90° = 0$ (leading)

R-L Series Circuit



In this ckt resistance R ohms and an inductive coil of inductance L henries are connected in series.

Let $V = V_m \sin \omega t$ be the applied voltage

i = ckt ct at any instant I = Effective value of ckt ct.

V_R = Potential diff. across Resistor V_L = Potential diff. across inductor

f = frequency of applied voltage

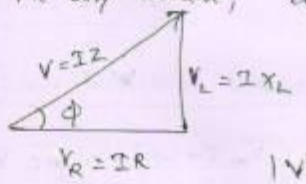
The same ct I flows through R and L.

I is taken as reference vector.

Voltage across R, $\bar{V}_R = IR$ in phase with I.

Voltage across L, $\bar{V}_L = I X_L$ leading I by $90°$

At any instant, applied voltage $V = V_R + V_L$



$V = IR + I X_L = I(R + jX_L)$

$V/I = R + jX_L = Z$ (impedance)

$|V| = \sqrt{V_R^2 + V_L^2} = \sqrt{(IR)^2 + (IX_L)^2} = \sqrt{I^2(R^2 + X_L^2)}$

$= I\sqrt{R^2 + X_L^2}$

$I = \frac{V}{\sqrt{R^2 + X_L^2}}$

$Z = \frac{V}{I} = \sqrt{R^2 + X_L^2}$

$\tan\phi = \frac{V_L}{V_R} = \frac{IX_L}{IR}$

$\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$

Power factor : (i) $\cos\phi = V_R / V = IR/IZ = R/Z$

(ii) $\cos\phi = \cos\left[\tan^{-1}(X_L/R)\right]$.

Power Calculation

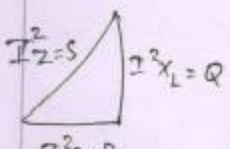
Real Power, $P = VI\cos\phi = VI\left(R/Z\right)$

$= \frac{V}{Z} \times I \times R = I^2R$ (Watts).

Reactive Power, $Q = VI\sin\phi = VI\left(\frac{X_L}{Z}\right)$

$= \frac{V}{Z}\, I\, X_L = I^2 X_L$ (VAR).

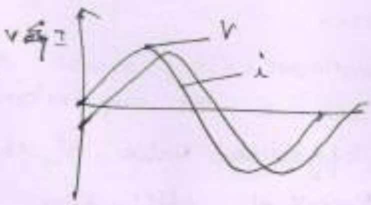
Complex power, $S = V \times I = I^2 Z = \frac{V^2}{Z}$



$\bar{S} = P + jQ$

$S = \sqrt{P^2+Q^2}$

Waveform :

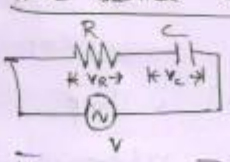


$v = V_m \sin\omega t$

$i = I_m \sin(\omega t - \phi)$

$I_m = \frac{V_m}{Z}$

R-C series ckt



In this ckt, Resistance R Ω & a pure capacitance of C Farad are connected in series.

$\bar{V}_R = \bar{I}R$; $\bar{V}_C = \bar{I}X_C$; Applied voltage $\bar{V} = \bar{V}_R + \bar{V}_C$

$\bar{V} = \bar{I}R - j\bar{I}X_C$

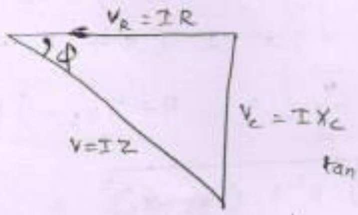
$\bar{V} = \bar{I}(R - jX_C)$

$\bar{Z} = V/I = R - jX_C$

$|V| = \sqrt{V_R^2 + (-V_C)^2} = \sqrt{(IR)^2 + (IX_C)^2} = I\sqrt{R^2+X_C^2}$

$I = \frac{V}{\sqrt{R^2+X_C^2}}$

$Z = V/I = \sqrt{R^2+X_C^2}$



$\tan\phi = \frac{IX_C}{IR} = \frac{X_C}{R} = \frac{1}{\omega CR}$,

$\phi = \tan^{-1}(X_C/R)$

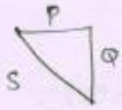
$\phi = \tan^{-1}\left(\frac{1}{\omega CR}\right)$

Power factor

(i) $\cos\phi = R/Z$

(ii) $\cos\phi = \cos\left(\tan^{-1}\left(\frac{X_C}{R}\right)\right)$

Power Calculation :



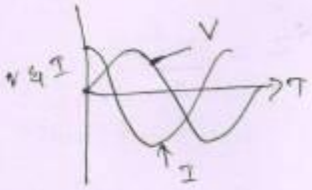
Real Power, $P = VI\cos\phi$

Reactive Power, $Q = VI\sin\phi$

Apparent Power, $\bar{S} = P + jQ$

$S = \sqrt{P^2 + Q^2}$

Waveform

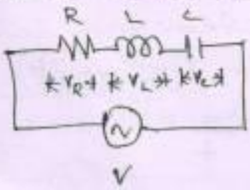


$v = V_m \sin\omega t$;

$i = I_m \sin(\omega t + \phi)$

$I_m = \frac{V_m}{2}$

R-L-C series ckt :



$v = V_m \sin\omega t$

i - ckt ct at any instant

I - Effective value of ckt ct

V_R - Potential difference across Resistor

V_L - " " Inductor

V_C - " " Capacitor

f - frequency of applied voltage

Same ct

I flows through R, L, C. I is taken as reference vector

$\bar{V}_R = IR$ in phase with ct.

$\bar{V}_L = IX_L \angle 90^\circ$ ∵ voltage leads ct by 90°

$\bar{V}_C = IX_C \angle -90^\circ$ ∵ voltage lags ct by 90°

Applied voltage $\bar{V} = \bar{V}_R + \bar{V}_L + \bar{V}_C$

$= IR + jIX_L - jIX_C$

$= I(R + jX_L - jX_C)$

$$\bar{Z} = \frac{\bar{V}}{\bar{I}} = R + jX_L - jX_C = R + j(X_L - X_C)$$

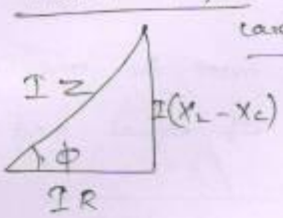
$$|\bar{Z}| = \sqrt{R^2 + (X_L - X_C)^2}$$

$X_L - X_C = X$ is called net reactance. If $X_L > X_C$ the ckt behave like R-L ckt. If $X_C > X_L$, the ckt will behave like R-C ckt.

$$|V| = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$I = V / \sqrt{R^2 + (X_L - X_C)^2}$$

Phasor diagram



case (i) : $X_L > X_C$

X will be inductive in nature.

$$\tan\phi = \frac{X_L - X_C}{R} = \frac{\omega L - \frac{1}{\omega C}}{R}$$

$$\phi = \tan^{-1}\left(\frac{\omega L - 1/\omega C}{R}\right)$$

$$\cos\phi = R/Z$$

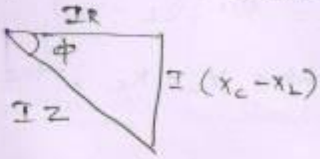
Current lags applied voltage by an angle ϕ.

$\therefore$ p.f is lagging.

case (ii) $X_C > X_L$

X will be capacitive in nature if $X_C > X_L$.

$\therefore$ the ckt behave like a R-C ckt.



$$\tan\phi = \frac{X_C - X_L}{R} = \frac{1/\omega C - \omega L}{R}$$

$$\phi = \tan^{-1}\left(\frac{(1/\omega C) - \omega L}{R}\right)$$

power factor, $\cos\phi = R/Z$

Current leads applied voltage by an angle ϕ.

$\therefore$ p.f is lead.

Power Calculation :

$P = VI\cos\phi$ Watts

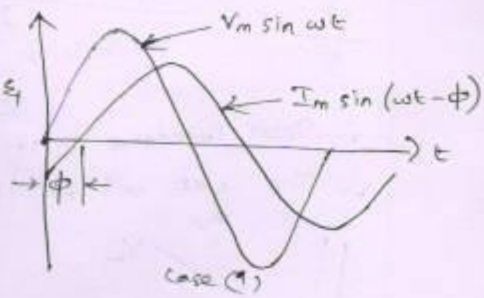
$Q = VI\sin\phi$ VAR

$S = VI$ volt Amp

$S = \sqrt{P^2 + Q^2}$

$\cos\phi = P/S$

Waveform

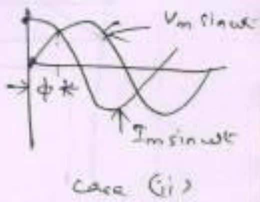


$V = V_m \sin \omega t$

$i = I_m \sin(\omega t \pm \phi)$

+ for case (ii) i.e $X_C > X_L$

- for case (i) i.e $X_L > X_C$

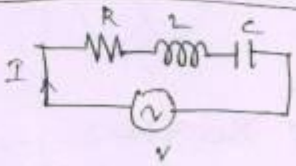


Resonance :

For any LC combination n/w, there must be one fre[quency] at which X_L equals X_C. This case of equal and oppo[site] reactances is called resonance.

The frequency at which the two reactances are equal is called as resonant frequency.

Resonance in Series AC Ckts :



Applied Voltage $\bar{V} = \bar{V}_R + \bar{V}_L + \bar{V}_C$

$$\bar{V} = \bar{I}R + j\bar{I}X_L - j\bar{I}X_C$$

$$\bar{V} = \bar{I}\,(R + jX_L - jX_C)$$

Total impedance, $\bar{Z} = \dfrac{\bar{V}}{\bar{I}} = R + j(X_L - X_C)$

At a certain frequency, $X_L = X_C$. When $X_L = X_C$, the ckt is said to be in resonance.

Resonant frequency :

$$X_L = X_C$$

$$2\pi f_r L = \frac{1}{2\pi f_r C}$$

$$f_r^2 = \frac{1}{4\pi^2 LC} \quad ; \quad f_r = \frac{1}{2\pi\sqrt{LC}}.$$

In a series R-L-C series ckt,

$$\bar{Z} = R + j(X_L - X_C)$$

$$= R + j\,\underbrace{\cancel{(X_L - X_L)}}_{0} \quad (\because X_L = X_C)$$

$$Z = R$$

∴ Total impedance Z is minimum & is equal to R.

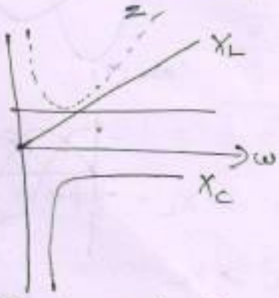
∴ The ckt will be purely resistive ckt.

$$P.f = \cos\phi = 1$$

Current $I = V/Z$ = maximum I_0

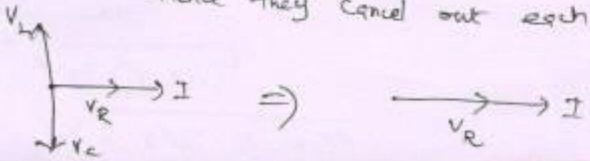
$$V_L = IX_L \quad ; \quad V_C = IX_C$$

Since ($X_L = X_C$) these two voltages (V_L & V_C) are equal in magnitude & opposite in phase.

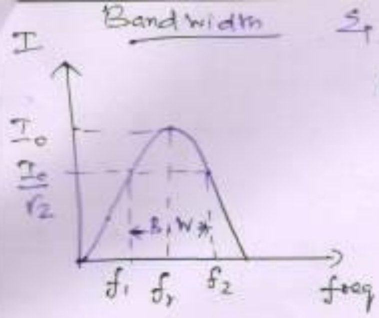


Impedance Curve

Hence they cancel out each other.



Bandwidth & Q factor :



When $f < f_r$, X_c will be greater than X_L

$\therefore$ p.f is leading.

When $f > f_r$, X_L will be greater than X_c.

$\therefore$ p.f is lagging.

Quality factor : It is defined as the voltage magnification at resonance.

$$Q\text{ factor} = \frac{V_c}{V} = \frac{I_o X_c}{I_o R} = \frac{X_c}{R} = \frac{1}{\omega_r C R}$$

$$Q = \frac{X_L}{R} = \frac{\omega_r L}{R}$$

$$Q = \frac{2\pi f_r L}{R} = \frac{\frac{1}{2\pi\sqrt{LC}} 2\pi L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}.$$

Bandwidth Bandwidth of a ckt is gn by the band of frequencies which lies b/w two points on either side of the resonant frequency where ct falls to $1/\sqrt{2}$ of its maximum value.

$$\text{Band width} = \Delta f = f_2 - f_1$$

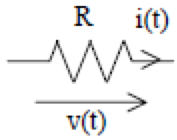
$$B.W = \frac{\text{Resonant frequency}}{Q\text{ factor}} = \frac{f_r}{Q}$$

Network Theorems

The fundamental laws that govern electric circuits are the Ohm's Law and the Kirchoff's Laws.

Ohm's Law

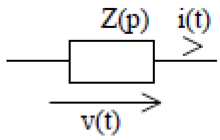
Ohm's Law states that the voltage $v(t)$ across a resistor R is directly proportioal to the current $i(t)$ flowing through it.



$$v(t) \propto i(t)$$

or $$v(t) = R \, . \, i(t)$$

This general statement of Ohm's Law can be extended to cover inductances and capacitors as well under alternating current conditions and transient conditions. This is then known as the Generalised Ohm's Law. This may be stated as



$$v(t) = Z(p) \, . \, i(t) \, ,$$ where p = d/dt = differential operator

$Z(p)$ is known as the impedance function of the circuit, and the above equation is the differential equation governing the behaviour of the circuit.

For a resistor, $Z(p) = R$

For an inductor $Z(p) = L\,p$

For a capacitor, $Z(p) = \dfrac{1}{C\,p}$

In the particular case of alternating current, $p = j\omega$ so that the equation governing circuit behaviour may be written as

$$V = Z(j\omega) \, . \, I \, ,$$ and

For a resistor, $Z(j\omega) = R$

For an inductor $Z(j\omega) = j\omega\,L$

For a capacitor, $Z(j\omega) = \dfrac{1}{j\omega\,C}$

We cannot analyse electric circuits using Ohm's Law only. We also need the Kirchoff's current law and the Kirchoff's voltage law.

Kirchoff's Current Law

Kirchoff's current law is based on the principle of conservation of charge. This requires that the algebraic sum of the charges within a system cannot change. Thus the total rate of change of charge must add up to zero. Rate of change of charge is current.

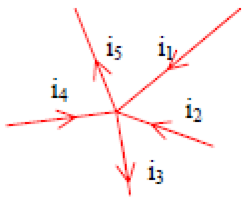
This gives us our basic Kirchoff's current law as the algebraic sum of the currents meeting at a point is zero.

i.e. at a node, $\Sigma\, I_r = 0$, where I_r are the currents in the branches meeting at the node

This is also sometimes stated as the sum of the currents entering a node is equal to the sum of the current leaving the node.

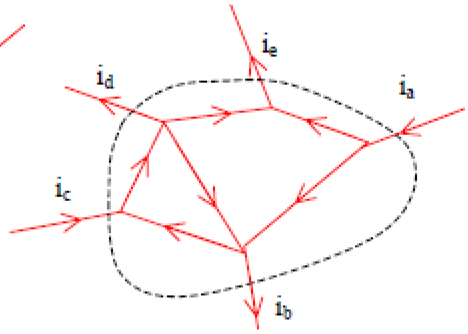
The theorem is applicable not only to a node, but to a closed system.

$$i_1 + i_2 - i_3 + i_4 - i_5 = 0$$
$$i_1 + i_2 + i_4 = i_3 + i_5$$



Also for the closed boundary,

$$i_a - i_b + i_c - i_d - i_e = 0$$



Kirchoff's Voltage Law

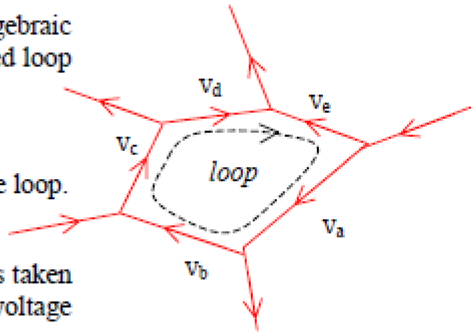
Kirchoff's voltage law is based on the principle of conservation of energy. This requires that the total work done in taking a unit positive charge around a closed path and ending up at the original point is zero.

This gives us our basic Kirchoff's law as the algebraic sum of the potential differences taken round a closed loop is zero.

i.e. around a loop, $\Sigma V_r = 0$,

where V_r are the voltages across the branches in the loop.

$$v_a + v_b + v_c + v_d - v_e = 0$$

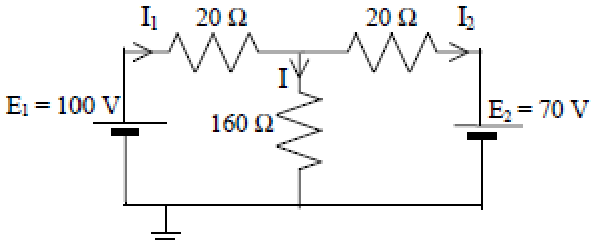


This is also sometimes stated as the sum of the emfs taken around a closed loop is equal to the sum of the voltage drops around the loop.

Although all circuits could besolved using only Ohm's Law and Kirchoff's laws, the calculations would be tedious. Various network theorems have been formulated to simplify these calculations.

Example 1

For the purposes of understanding the principle of the Ohm's Law and the Kirchoff's Laws and their applicability, we will consider only a resistive circuit. However it must be remembered that the laws are applicable to alternating currents as well.



For the circuit shown in the figure, let us use Ohm's Law and Kirchoff's Laws to solve for the current I in the 160 Ω resistor.

Using Kirchoff's current law

$$I = I_1 - I_2$$

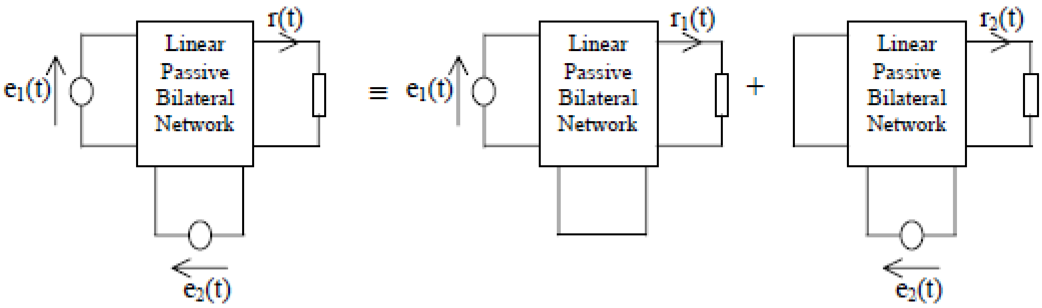
Using Kirchoff's voltage law

$$100 = 20\, I_1 + 160\,(I_1 - I_2) \quad \Rightarrow \quad 10 = 18\, I_1 - 16\, I_2$$
$$-70 = 20\, I_2 - 160\,(I_1 - I_2) \quad \Rightarrow \quad 7 = 16\, I_1 - 18\, I_2$$

which has the solution $I_1 = 1$ A, $I_2 = 0.5$ A and the unknown current $I = 0.5$A.

Superposition Theorem

The superposition theorem tells us that if a network comprises of more than one source, the resulting currents and voltages in the network can be determined by taking each source independently and superposing the results.



If an excitation $e_1(t)$ alone gives a response $r_1(t)$,

and an excitation $e_2(t)$ alone gives a response $r_2(t)$,

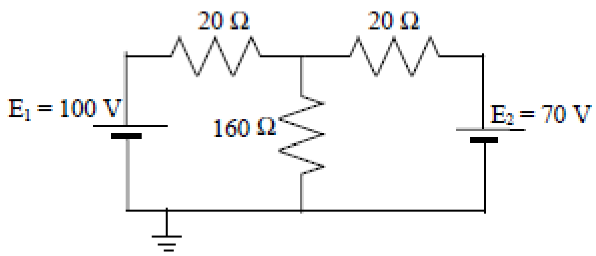
then, by superposition theorem, if the excitation $e_1(t)$ and the excitation $e_2(t)$ together would give a response $r(t) = r_1(t) + r_2(t)$

The superposition theorem can even be stated in a more general manner, where the superposition occurs with scaling.

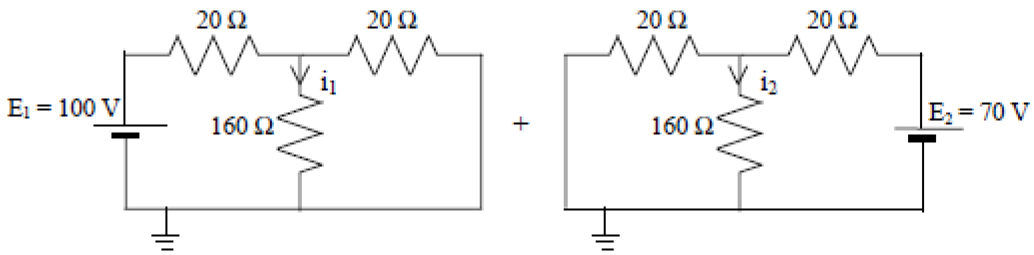
Thus an excitation of $k_1\, e_1(t)$ and an excitation of $k_2\, e_2(t)$ occuring together would give a response of $k_1\, r_1(t) + k_2\, r_2(t)$.

Example 2

Let us solve the same problem as earlier, but using Superposition theorem.



Solution



$$\text{for circuit 1, source current} = \frac{100}{20 + 160//20} = \frac{100}{20 + \dfrac{160 \times 20}{180}} = \frac{100}{37.778} = 2.647\,A$$

$$\therefore i_1 = 2.647 \times \frac{20}{180} = 0.294\text{ A}$$

Similarly for circuit 2, source current $= \dfrac{70}{20+160//20} = \dfrac{70}{20+\dfrac{160\times 20}{180}} = \dfrac{70}{37.778} = 1.853\,A$

$\therefore i_2 = 1.853 \times \dfrac{20}{180} = 0.206$ A

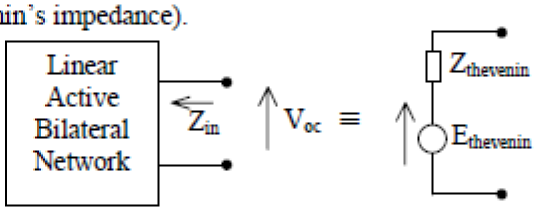
$\therefore$ unknown current $i = i_1 + i_2 = 0.294 + 0.206 = 0.500$ A

which is the same answer that we got from Kirchoff's Laws and Ohm's Law.

Thevenin's Theorem (or Helmholtz's Theorem)

The Thevenin's theorem, basically gives the equivalent voltage source corresponding to an active network.

If a linear, active, bilateral network is considered across one of its ports, then it can be replaced by an equivalent voltage source (Thevenin's voltage source) and an equivalent series impedance (Thevenin's impedance).



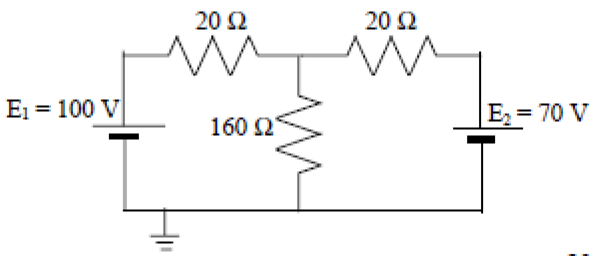
Since the two sides are identical, they must be true for all conditions. Thus if we compare the voltage across the port in each case under open circuit conditions, and measure the input impedance of the network with the sources removed (voltage sources short-circuited and current sources open-circuited), then

$E_{thevenin} = V_{oc}$, and

$Z_{thevenin} = Z_{in}$

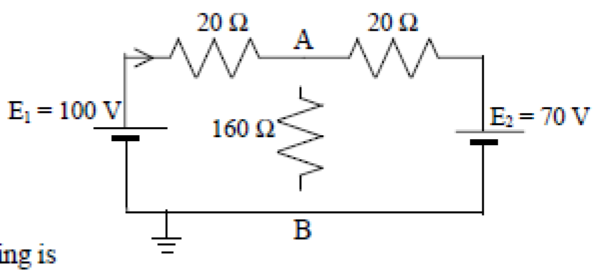
Example 2

Let us again consider the same example to illustrate Thevenin's Theorem.



Solution

Since we wish to calculate the current in the 160 Ω resistor, let us find the Thevenin's equivalent circuit across the terminals after disconnecting (open circuiting) the 160 Ω resistor.



Under open circuit conditions, current flowing is

$= (100 - 70)/40 = 0.75$ A

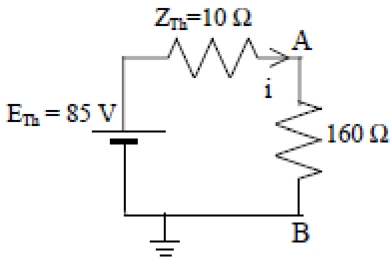
$\therefore \quad V_{oc,AB} = 100 - 0.75 \times 20 = 85$ V

$\therefore\ E_{Th} = 85$ V

The input impedance across AB (with sources removed) = 20//20 = 10 Ω.

$\therefore\ Z_{Th} = 10\ \Omega$.

Therefore the Thevenin's equivalent circuit may be drawn with branch AB reintroduced as follows.



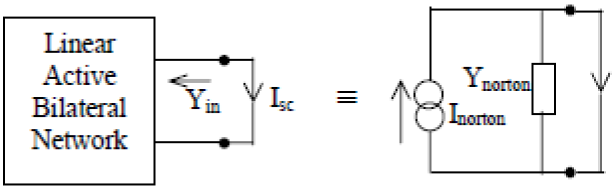
From the equivalent circuit, the unknown current i is determined as

$$i = \frac{85}{10+160} = 0.5A$$

which is the same result that was obtained from the earlier two methods.

Norton's Theorem

Norton's Theorem is the dual of Thevenin's theorem, and states that any linear, active, bilateral network, considered across one of its ports, can be replaced by an equivalent current source (Norton's current source) and an equivalent shunt admittance (Norton's Admittance).



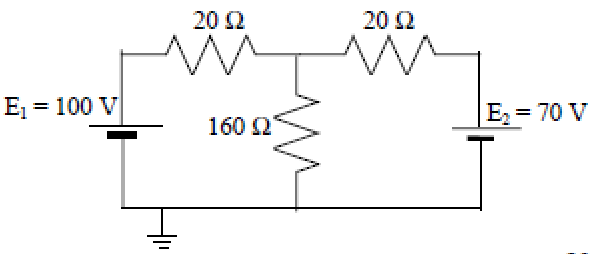
Since the two sides are identical, they must be true for all conditions. Thus if we compare the current through the port in each case under short circuit conditions, and measure the input admittance of the network with the sources removed (voltage sources short-circuited and current sources open-circuited), then

$I_{norton} = I_{sc}$, and

$Y_{norton} = Y_{in}$

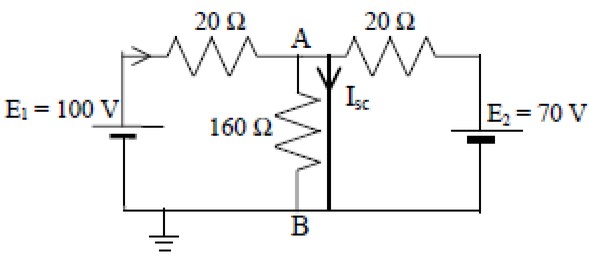
Example 3

Let us again consider the same example to illustrate Norton's Theorem.



Solution

Since we wish to calculate the current in the 160 Ω resistor, let us find the Norton's equivalent circuit across the terminals after short-circuiting the 160 Ω resistor.



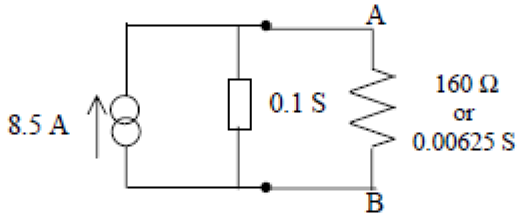
the short circuit current I_{sc} is given by

$I_{sc} = 100/20 + 70/20 = 8.5$ A

$\therefore \quad I_{norton} = 8.5$ A

Norton's admittance = 1/20 + 1/20 = 0.1 S

∴ Norton's equivalent circuit is



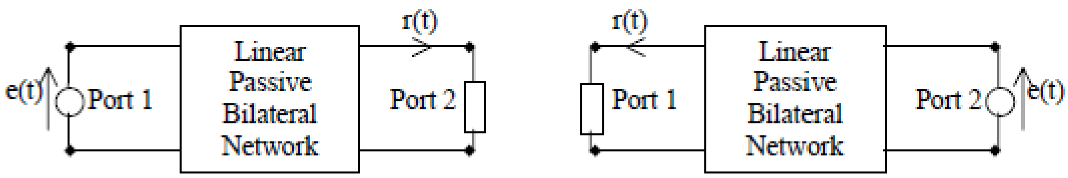
and the current in the unknown resistor is $8.5 \times \dfrac{0.00625}{0.1 + 0.00625} = 0.5\,A$

which is the same result as before.

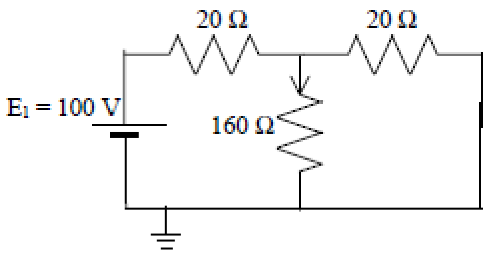
Reciprocality Theorem

The reciprocality theorem tells us that in a linear passive bilateral network an excitation and the corresponding response may be interchanged.

In a two port network, if an excitation e(t) at port (1) produces a certain response r(t) at a port (2), then if the same excitation e(t) is applied instead to port (2), then the same response r(t) would occur at the other port (1).

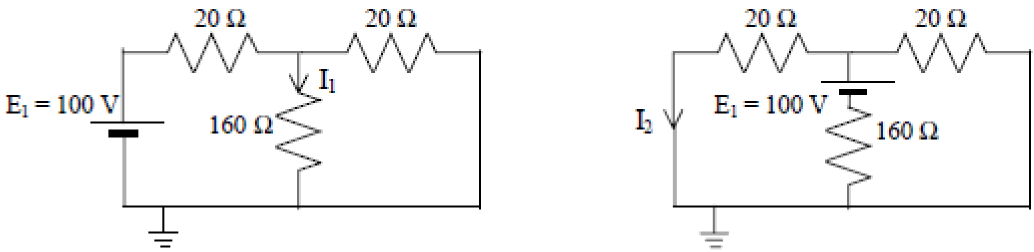


Example 4



Consider the earlier example, but with only one source. Determine the current in the 160 Ω branch. Now replace the 160 Ω resistor with the source in series with it and after short-circuit the source at the original location, find the current flowing at the original source location. Show that it verifies the Reciprocality theorem.

Solution



For the original circuit, current $I_1 = \dfrac{100}{20 + 160 // 20} \times \dfrac{20}{20 + 160} = \dfrac{2000}{37.778 \times 180} = 0.294\,A$

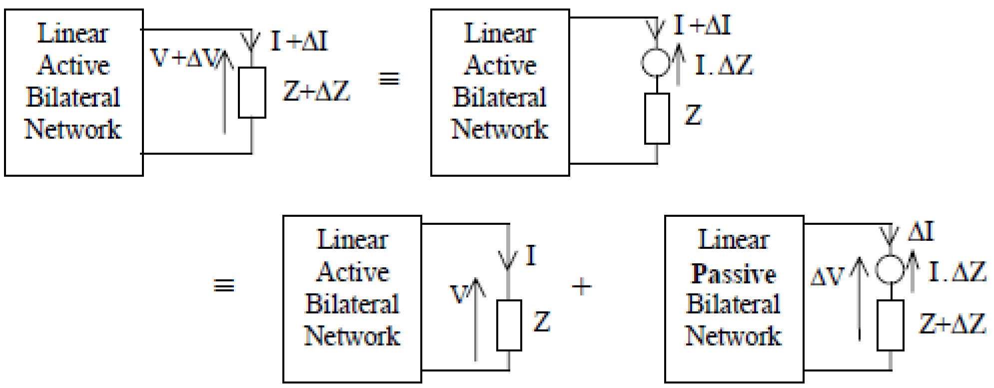
similarly for the new circuit, current $I_2 = \dfrac{100}{160+20//20} \times \dfrac{20}{20+20} = \dfrac{2000}{170 \times 40} = 0.294\,A$

It is seen that the identical current has appeared verifying the reciprocality theorem. The advantage of the theorem is when a circuit has already been analysed for one solution, it may be possible to find a corresponding solution without further work.

Compensation Theorem

In many circuits, after the circuit is analysed, it is realised that only a small change need to be made to a component to get a desired result. In such a case we would normally have to recalculate. The compensation theorem allows us to compensate properly for such changes without sacrificing accuracy.

In any linear bilateral active network, if any branch carrying a current I has its impedance Z changed by an amount ΔZ, the resulting changes that occur in the other branches are the same as those which would have been caused by the injection of a voltage source of (-) I . ΔZ in the modified branch.



Consider the voltage drop across the modified branch.

$$V+\Delta V = (Z+\Delta Z)(I+\Delta I) = Z\,.\,I + \Delta Z\,.\,I + (Z+\Delta Z)\,.\,\Delta I$$

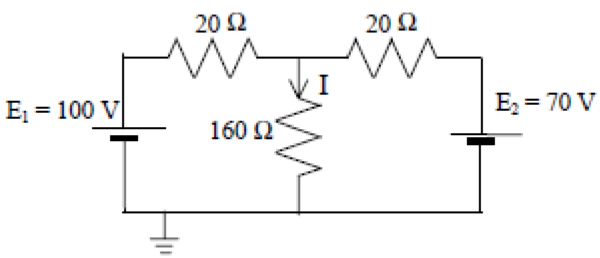
from the original network, $V = Z\,.\,I$

$$\therefore \quad \Delta V = \Delta Z\,.\,I + (Z+\Delta Z)\,.\,\Delta I$$

Since the value I is already known from the earlier analysis, and the change required in the impedance, ΔZ , is also known, I .ΔZ is a known fixed value of voltage and may thus be represented by a source of emf I. ΔZ .

Using superposition theorem, we can easily see that the original sources in the active network give rise to the original current I, while the change corresponding to the emf I. ΔZ must produce the remaining changes in the network.

Example 5



From example 4, we saw that the current in the 160 Ω resistor is 0.5 A.

Let us say that we want to change the resistor by a quantity ΔR such that the current in the 160 Ω resistor is 0.600 A. Then the circuit for changes can be written as

$\Delta I = 0.6 - 0.5 = 0.1$ A

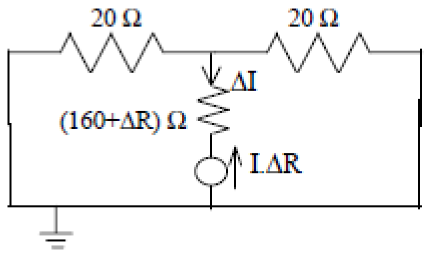
$I = 0.5$

$$\therefore \quad \Delta I = \frac{(-)0.5 \times \Delta R}{160 + \Delta R + 20//20}$$

$$\text{i.e. } 0.1 = (-)\frac{0.5 \times \Delta R}{170 + \Delta R}$$

$\therefore 17 + 0.1\,\Delta R = (-)\,0.5\,\Delta R$

i.e. $\Delta R = (-)17/0.6 = (-)\,28.333\ \Omega$



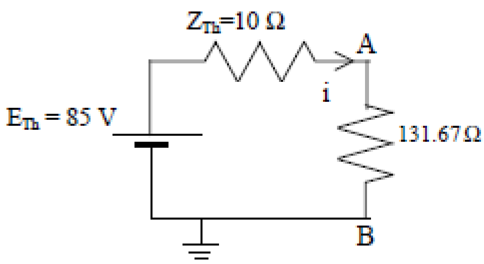
Therefore the required value of R = 160 – 28.333 = 131.67 Ω

This could have been calculated using Kirchoff's and Ohm's laws but would have been more complicated.

We can also check this answer with Thevenin's theorem as follows.

From Example 2, we had the Thevenin's circuit as shown, with the 160 Ω replaced by 131.67 Ω.

The current for this value can be quickly obtained as $i = \dfrac{85}{10 + 131.667} = 0.6\,A$



So you can also see that by knowing Thevenin's equivalent circuit for a given network, we can obtain solutions for many conditions with little additional calculations.

The same is true with Norton's theorem.

Maximum Power Transfer Theorem

As you are probably aware, a normal car battery is rated at 12 V and generally has an open circuit voltage of around 13.5 V. Similarly, if we take 7 pen-torch batteries, they too will have a terminal voltage of $7 \times 1.5 = 13.5$ V. However, you would also be aware, that if your car battery is dead, you cannot go to the nearest shop, buy 7 pen-torch batteries and start your car. Why is that ? Because the pen-torch batteries, although having the same open circuit voltage does not have the necessary power (or current capacity) and hence the required current could not be given. Or if stated in different terms, it has too high an internal resistance so that the voltage would drop without giving the necessary current.

This means that a given battery (or any other energy supply, such as the mains) can only give a limited amount of power to a load. The maximum power transfer theorem defines this power, and tells us the condition at which this occurs.

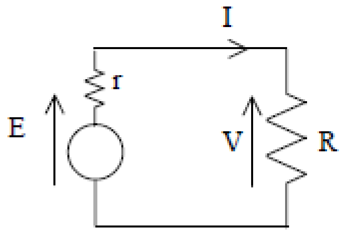
For example, if we consider the above battery, maximum voltage would be given when the current is zero, and maximum current would be given when the load is short-circuit (load voltage is zero). Under both these conditions, there is no power delivered to the load. Thus obviously in between these two extremes must be the point at which maximum power is delivered.

The Maximum Power Transfer theorem states that for maximum active power to be delivered to the load, load impedance must correspond to the conjugate of the source impedance (or in the case of direct quantities, be equal to the source impedance).

Let us analyse this, by first starting with the basic case of a resistive load being supplied from a source with only an internal resistance (this is the same as for d.c.)

Resistive Load supplied from a source with only an internal resistance

Consider a source with an internal emf of E and an internal resistance of r and a load of resistance R.



current I $= \dfrac{E}{R+r}$

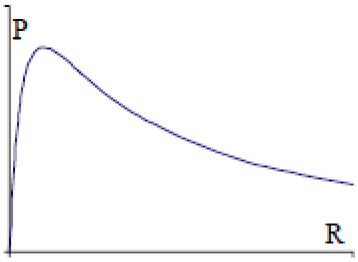
Load Power P = I^2. R $= \left(\dfrac{E}{R+r}\right)^2 \cdot R$

The source resistance is dependant purely on the source and is a constant, as is the source emf. Thus only the load resistance R is a variable.

To obtain maximum power transfer to the load, let us differentiate with respect to R.

$$\frac{dP}{dR} = \frac{E^2}{(R+r)^4} \cdot \left[(R+r)^2 \cdot 1 - R \cdot 2(R+r)\right] = 0 \text{ for maximum}$$

[Note: I said maximum, rather than maximum or minimum, because from physical considerations we know that there must a maximum power in the range. So we need not look at the second derivative to see whether it is maximum or minimum].



$\therefore\ (R+r)^2 - 2R\,(R+r) = 0$

or $R + r - 2R = 0$

or $R + r - 2R = 0$

i.e. R = r for maximum power transfer.

value of maximum power = $P_{max} = \left(\dfrac{E}{r+r}\right)^2 \cdot r = \dfrac{E^2}{4r}$

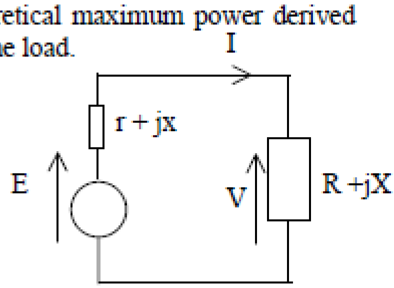
load voltage at maximum power = $\dfrac{E}{R+r}.R = \dfrac{E}{r+r}.r = \dfrac{E}{2}$

It is to be noted that when maximum power is being transferred, only half the applied voltage is available to the load, and the other half drops across the source. Also, under these conditions, half the power supplied is wasted as dissipation in the source.

Thus the useful maximum power will be less than the theoretical maximum power derived and will depend on the voltage required to be maintained at the load.

Load supplied from a source with an internal impedance

Consider a source with an internal emf of E and an internal impedance of z = (r + j x) and a load of impedance Z = R + jX.



current I $= \dfrac{E}{r + jx + R + jX} = \dfrac{E}{(r+R) + j(x+X)}$

magnitude of $\mathrm{I} = \dfrac{E}{\sqrt{(r+R)^2 + (x+X)^2}}$

Load Power P = $|\mathrm{I}|^2 . \mathrm{R} = \dfrac{E^2}{(r+R)^2 + (x+X)^2} \cdot R$

Since there are two variables R and X, for maximum power $\dfrac{\partial P}{\partial R} = 0 \quad and \quad \dfrac{\partial P}{\partial X} = 0$

i.e. $\dfrac{E^2}{\left[(r+R)^2 + (x+X)^2\right]^2} \cdot \left[\left[(r+R)^2 + (x+X)^2\right] \cdot 1 - R[2(r+R)]\right] = 0$

and $\dfrac{E^2}{\left[(r+R)^2 + (x+X)^2\right]^2} \cdot \left[-R \cdot 2 \cdot (x+X)\right]$

The second equation gives $x + X = 0$ or $X = -x$

Substituting this in the first equation gives $(r+R)^2 - R \,.\, 2(r+R) = 0$

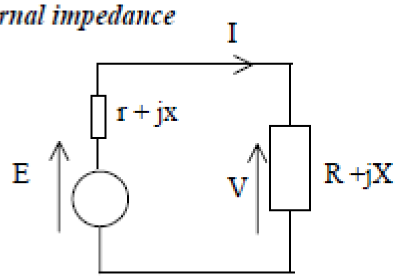
Since R cannot be negative, $r + R \neq 0$. $\therefore r + R - 2R = 0$

i.e. $\mathrm{R} = \mathrm{r}$

$\therefore$ $\mathrm{Z} = \mathrm{R} + \mathrm{j}\,\mathrm{X} = \mathrm{r} - \mathrm{j}\,\mathrm{x} = \mathrm{z}^*$

Therefore for maximum power transfer, the load impedance must be equal to the conjugate of the source impedance.

Load of fixed power factor supplied from a source with an internal impedance



Consider a source with an internal emf of E and an internal impedance of $z = (r + j\,x)$ and a load of impedance $Z = R + jX$ which has a given power factor k. [This situation is not uncommon, as for example if the load was an induction motor load, the power factor would have a fixed value such as 0.8 lag]

current I $= \dfrac{E}{r + jx + R + jX} = \dfrac{E}{(r+R) + j(x+X)}$

R and X are no longer independent but have the relationship $power\ factor\ f = \dfrac{R}{\sqrt{R^2 + X^2}}$

or $X = \sqrt{\dfrac{1}{f^2} - 1} \cdot R \; = k \,.\, R$

magnitude of $\mathrm{I} = \dfrac{E}{\sqrt{(r+R)^2 + (x+X)^2}} = \dfrac{E}{\sqrt{(r+R)^2 + (x+k.R)^2}}$

Load Power P = $|\mathrm{I}|^2 . \mathrm{R} = \dfrac{E^2}{(r+R)^2 + (x+k.R)^2} \cdot R$

Since there is only one variable R, for maximum power $\frac{dP}{dR} = 0$

i.e. $\quad \frac{E^2}{\left[(r+R)^2+(x+k.R)^2\right]^2}\cdot\left[\left[(r+R)^2+(x+k.R)^2\right].1 - R\left[2(r+R)+2(x+k.R).k\right]\right] = 0$

i.e. $\quad (r+R)^2 + (x+k.R)^2 - 2R(r+R) - 2k.R(x+k.R) = 0$

i.e. $\quad r^2 + \cancel{2r.R} + R^2 + x^2 + \cancel{2k.x.R} + k^2.R^2 - \cancel{2R.r} - 2R^2 - \cancel{2k.R.x} - 2k^2.R^2 = 0$

i.e. $\quad r^2 - R^2 + x^2 - k^2.R^2 = 0$

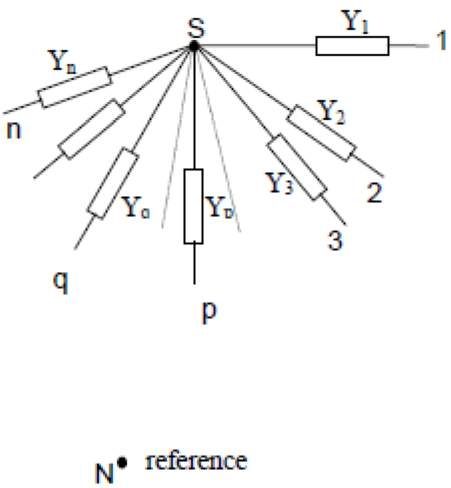
i.e. $\quad R^2 + k^2R^2 = r^2 + x^2$. $\qquad$ i.e. $R^2 + X^2 = r^2 + x^2$

i.e. $\quad |Z| = |z|$

So even when the power factor of the load is different from that of the source, a condition that needs to be satisfied is that the magnitude of the load impedance must be equal to the magnitude of the source impedance.

Note: If limits are placed on the voltage, then maximum power will not always occur under the above condition, but at the limit of the voltage closest to the desired solution.

Millmann's Theorem



Consider a number of admittances Y_1, Y_2, Y_3 ....Y_p.... Y_q, ...Y_n are connected together at a common point S. If the voltages of the free ends of the admittances with respect to a common reference N are known to be V_{1N}, V_{2N}, V_{3N} ....V_{pN}.... V_{qN}, ...V_{nN}, then Millmann's theorem gives the voltage of the common point S with respect to the reference N as follows.

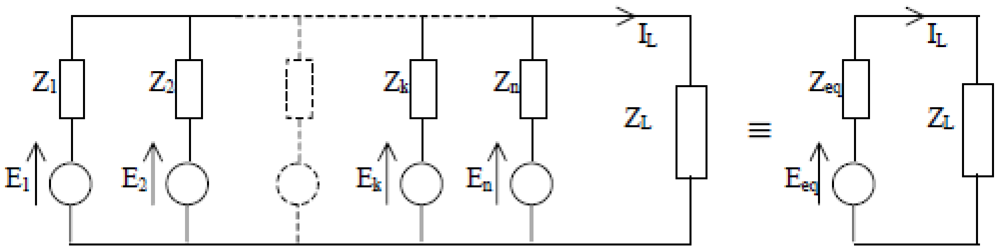
Applying Kirchoff's Current Law at node S

$\sum_{p=1}^{n} I_p = 0$, $I_p = Y_p\,(V_{pN} - V_{SN})$

i.e. $\quad \sum_{p=1}^{n} Y_p (V_{pN} - V_{SN}) = 0$,

i.e. $\sum_{p=1}^{n} Y_p V_{pN} = V_{SN} \sum_{p=1}^{n} Y_p \qquad$ so that $V_{SN} = \frac{\sum_{p=1}^{n} Y_p V_{pN}}{\sum_{p=1}^{n} Y_p}$

An extension of the Millmann theorem is the *equivalent generator theorem*.

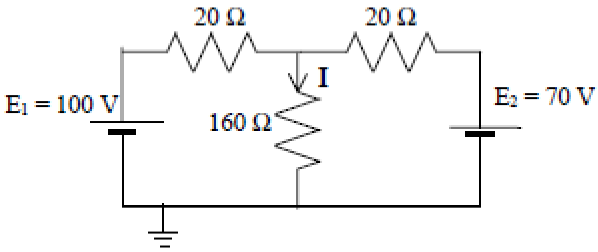


This theorem states that a system of voltage sources operating in parallel may be replaced by a single voltage source in series with an equivalent impedance given as follows (this is effectively the Thevenin's theorem applied to a number of generators in parallel).

$$E_{eq} = \frac{\sum_{k=1}^{n} E_k Y_k}{\sum_{k=1}^{n} Y_k}, \quad Y_{eq} = \sum_{k=1}^{n} Y_k$$

Example 6

The figure shown (also used in earlier examples) can be considered equivalent to two sources of 100 V and 70 V, with internal resistances 20 Ω each, feeding a load of 160 Ω. Using Millmann's theorem (or equivalent generator theorem) find the current I.



Solution

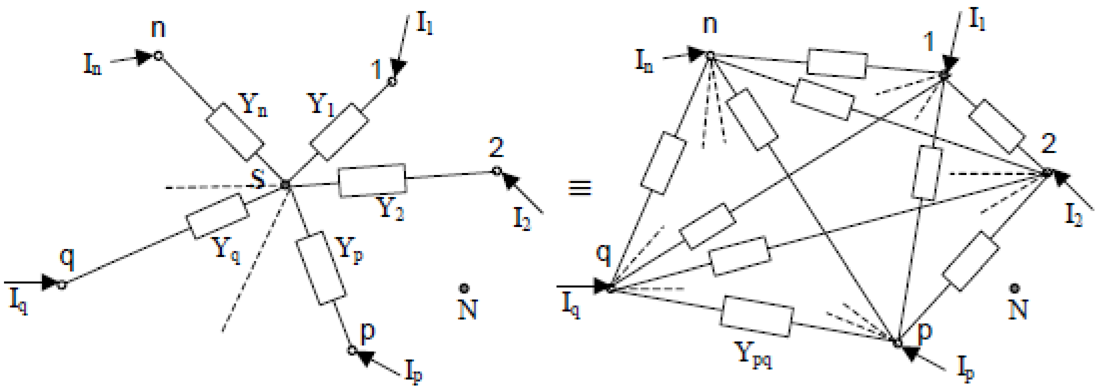
From Equivalent generator theorem, it has been shown that the equivalent generator has

$$E_{eq} = \frac{\frac{1}{20}100 + \frac{1}{20}70}{\frac{1}{20} + \frac{1}{20}} = 85 \text{ V} \quad \text{(same answer was obtained with Thevenin's Th}^{m}\text{)}$$

$$Z_{eq} = \frac{1}{\frac{1}{20} + \frac{1}{20}} = 10\,\Omega \quad \text{(again same answer was obtained with Thevenin's Th}^{m}\text{)}$$

Hence current $I = \frac{85}{10 + 160} = 0.5$ A

Rosen's Theorem (Nodal-Mesh Transformation Theorem)



Rosen's theorem tells us how we could find the mesh equivalent of a network where all the branches are connected to a single node. [In the mesh equivalent, all nodes are connected to each other and not to a common node as in the nodal network]. When the equivalent is obtained the external conditions are not affected as seen from the external currents in the above diagrams.

For the nodal network, from Millmann's theorem

$$V_{SN} = \frac{\sum_{p=1}^{n} Y_p V_{pN}}{\sum_{p=1}^{n} Y_p}, \text{ so that } \quad I_q = Y_q (V_{qN} - V_{SN}) = Y_q . V_{qN} - \frac{\sum_{p=1}^{n} Y_p V_{pN}}{\sum_{p=1}^{n} Y_p}$$

$$I_q = \frac{Y_q V_{qN} \sum_{p=1}^{n} Y_p - \sum_{p=1}^{n} Y_p V_{pN}}{\sum_{p=1}^{n} Y_p} = \frac{Y_q \sum_{p=1}^{n} Y_p (V_{qN} - V_{pN})}{\sum_{p=1}^{n} Y_p}$$

For a definite summation, whether the variable p is used or the variable k is used makes no difference. Thus

$$I_q = \frac{Y_q \sum_{p=1}^{n} Y_p (V_{qN} - V_{pN})}{\sum_{k=1}^{n} Y_k} = \sum_{p=1}^{n} \left[\frac{Y_p Y_q}{\sum_{k=1}^{n} Y_k} \right] (V_p - V_q)$$

For the mesh network, from Kirchoff's current law, the current at any node is

$$I_q = \sum_{p=1}^{n} Y_{pq} (V_p - V_q)$$

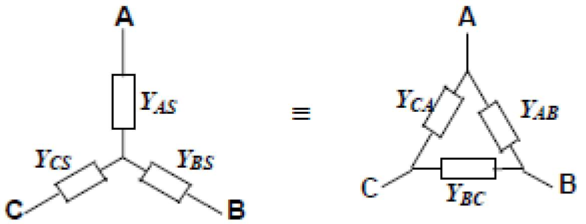
Comparing equations, it follows that a solution to equation is

$$Y_{pq} = \frac{Y_p Y_q}{\sum_{k=1}^{n} Y_k}$$ which is the statement of ***Rosen's theorem***

The converse of this theorem is in general not possible as there are generally more branches in the mesh network than in the nodal network.

However, in the case of the 3 node case, there are equal branches in both the nodal network (also known as star) and the mesh network (also known as delta).

Star-Delta Transformation



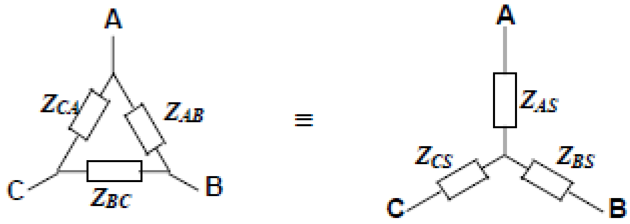
A *star connected* network of three admittances (or conductances) Y_{AS}, Y_{BS}, and Y_{CS} connected together at a common node S can be transformed into a *delta connected* network of three admittances Y_{AB}, Y_{BC}, and Y_{CA} using the following transformations. This has the same form as the general expression derived earlier.

$$Y_{AB} = \frac{Y_{AS} . Y_{BS}}{Y_{AS} + Y_{BS} + Y_{CS}}, \; Y_{BC} = \frac{Y_{BS} . Y_{CS}}{Y_{AS} + Y_{BS} + Y_{CS}}, \; Y_{CA} = \frac{Y_{CS} . Y_{AS}}{Y_{AS} + Y_{BS} + Y_{CS}}$$

Note: You can observe that in each of the above expressions if we need to find a particular delta admittance element value, we have to multiply the two values of admittance at the nodes on either side in the original star-network and divide by the sum of the three admittances.

In the special case of three nodes, reverse transformation is also possible.

Delta-Star Transformation



A *delta connected* network of three impedances (or resistances) Z_{AB}, Z_{BC}, and Z_{CA} can be transformed into a *star connected* network of three impedances Z_{AS}, Z_{BS}, and Z_{CS} connected together at a common node S using the following transformations. [You will notice that I have used impedance here rather than admittance because then the form of the solution remains similar and easy to remember.]

$$Z_{AS} = \frac{Z_{AB}.Z_{CA}}{Z_{AB} + Z_{BC} + Z_{CA}}, Z_{BS} = \frac{Z_{AB}.Z_{BC}}{Z_{AB} + Z_{BC} + Z_{CA}}, Z_{CS} = \frac{Z_{CA}.Z_{BC}}{Z_{AB} + Z_{BC} + Z_{CA}}$$

Note: You can observe that in each of the above expressions if we need to find a particular delta element value, we have to multiply the two impedance values on either side of node in the original star-network and divide by the sum of the three impedances.

Proof:

Impedance between A and C with zero current in B can be compared in the two networks as follows.

$$Z_{CA}//(Z_{AB} + Z_{BC}) = Z_{CS} + Z_{AS}$$

i.e.
$$\frac{Z_{CA}(Z_{AB} + Z_{BC})}{Z_{CA} + Z_{AB} + Z_{BC}} = Z_{CS} + Z_{AS}$$

similarly
$$\frac{Z_{AB}(Z_{BC} + Z_{CA})}{Z_{AB} + Z_{BC} + Z_{CA}} = Z_{AS} + Z_{BS}$$

and
$$\frac{Z_{BC}(Z_{CA} + Z_{AB})}{Z_{BC} + Z_{CA} + Z_{AB}} = Z_{BS} + Z_{CS}$$

elimination of variables from the above equations gives the desired results.